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OK-EXTENDIBLE FILTERS ON ω

ANDRÉS MILLÁN

Abstract. In this note we prove that every meager filter can be extended to an OK-point and that there are $2^{\mathfrak{c}}$-many nonmeager and null filters having OK-point extensions as well. These results generalize a construction by K. Kunen. Also, we notice that is consistent with ZFC that some measure zero filters cannot have OK-point extensions. Finally, we prove that despite of the fact that there exist $2^{\mathfrak{c}}$-many OK-points, its generic existence is independent of the axioms of ZFC.

1. Introduction

OK point ultrafilters were introduced by K. Kunen in [4] in order to prove that the remainder of the Stone-Čech compactification of ω is not homogeneous. Kunen constructed OK-points by using a system of infinite sets of ω with strong combinatorial properties. However, it was not clear for which kind of filters other than the cofinite filter that a similar construction could be performed. Also, it was shown in [4] that in ZFC, OK points are relatively abundant in the sense that there are $2^{\mathfrak{c}}$-many of them but, it was not obvious whether "small" filters could be extended to OK-points. The lack of interest about these issues could be attributed to the fact that papers [1], [2] and [6] had not yet been published and possibly those questions were not relevant at that time. This note can be considered as a first attempt to answer them.

Our notation and terminology is fairly standard. The cofinite filter will be denoted $\mathscr{F}_{\text{cof}} = \{A \subseteq \omega \colon |\omega \setminus A| < \omega\}$. Letters $\mathscr{F}$, $\mathscr{G}$ and $\mathscr{H}$ will always denote a filter containing $\mathscr{F}_{\text{cof}}$. Letters $\mathscr{U}$ and $\mathscr{V}$ will denote

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