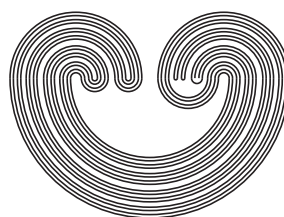


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BOXES AND NON-NORMALITY SETS

by

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BOXES AND NON-NORMALITY SETS

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ABSTRACT. We discuss normality and non-normality of spaces obtained by removing a box from the Cartesian product of spaces. Our results enable us, among others, to characterize non-normality points of Cartesian products of compact Hausdorff spaces.

1. INTRODUCTION

Preservation of normality by Cartesian products and their subspaces has been one of the most explored research topics in general topology. A vast literature exists on this subject (the reader may consult a bit outdated but still an excellent survey article by T. Przymusiński [8]). In our note, we discuss normality and non-normality of subspaces obtained by removing a box from a Cartesian product of spaces, i.e., subspaces of the form $\left(\prod_{s \in S} X_s\right) - B$, where $B = \prod_{s \in S} F_s$ and $F_s \subseteq X_s$ is closed for each $s \in S$. Two classical results, non-normality of the Tychonoff plank and non-normality of the space $\kappa \times (\kappa + 1)$ for any regular uncountable cardinal κ , are very special cases of our considerations.

A well known result of M. Katětov [4] asserts that if X and Y are compact Hausdorff spaces such that $X \times Y$ is hereditary normal, then the spaces X and Y are perfectly normal. We show that the same conclusion holds true assuming only that the subspaces of the form $X \times Y - E \times F$ are normal for all closed sets $E \subseteq X$ and $F \subseteq Y$. In fact, we show the converse statement to be true as well (see Theorem 6).

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