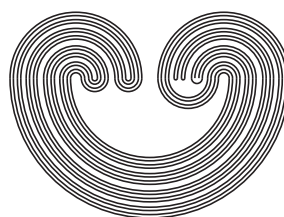


<http://topology.auburn.edu/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 47, 2016

Pages 273–278

---

<http://topology.nipissingu.ca/tp/>

## REMARKS ON METRIZABILITY OF DUAL GROUPS

by

FRÉDÉRIC MYNARD AND MIKHAIL TKACHENKO

Electronically published on October 21, 2015

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.



## REMARKS ON METRIZABILITY OF DUAL GROUPS

FRÉDÉRIC MYNARD AND MIKHAIL TKACHENKO

ABSTRACT. We examine sufficient conditions for the dual of a topological group to be metrizable and locally compact, improving on results of [4].

### 1. INTRODUCTION AND PRELIMINARIES

If  $G$  is an abelian topological group, its dual  $\widehat{G}$  is the set of continuous group homomorphisms into the one-dimensional torus  $\mathbb{T}$ , endowed with the compact-open topology.

Two families  $\mathcal{A}$  and  $\mathcal{B}$  of subsets of a set  $X$  are said to *mesh*, in symbols  $\mathcal{A}\#\mathcal{B}$ , if  $A \cap B \neq \emptyset$  whenever  $A \in \mathcal{A}$  and  $B \in \mathcal{B}$ . Thus, if the set  $\mathbb{F}X$  of filters on a set  $X$  is ordered by inclusion, two filters  $\mathcal{F}$  and  $\mathcal{G}$  admit a supremum  $\mathcal{F} \vee \mathcal{G}$  if and only if  $\mathcal{F}\#\mathcal{G}$ . We do not distinguish between a sequence  $(x_n)_{n \in \omega}$  and the filter generated by its tails. In particular, if  $\mathcal{H}$  is a family of subsets of  $X$  and  $(x_n)_{n \in \omega}$  is a sequence on  $X$ , the notation  $\mathcal{H}\#(x_n)_{n \in \omega}$  means that  $H \cap \{x_n : n \geq k\} \neq \emptyset$  for every  $H \in \mathcal{H}$  and  $k \in \omega$ .

Recall that a topological space  $X$  is *sequential* if every sequentially closed subset is closed. The space is *Fréchet-Urysohn*, or simply *Fréchet*, if whenever  $x \in X$  and  $A \subset X$  with  $x \in \text{cl } A$ , there is a sequence on  $A$  that converges to  $x$ . It is *strongly Fréchet* if whenever  $(A_n)_{n \in \omega}$  is a decreasing sequence of subsets of  $X$  with  $x \in \bigcap_{n \in \omega} \text{cl } A_n$  there is  $x_n \in A_n$  for each  $n$  with  $x_n \rightarrow_n x$ , equivalently if

$$\forall \mathcal{H} \in \mathbb{F}_1 X, \text{adh } \mathcal{H} \subseteq \text{adh}_{\text{Seq}} \mathcal{H},$$

---

2010 *Mathematics Subject Classification*. 54H11; 54A20; 54C35; 22A05.

*Key words and phrases*. Topological group; metrizability; Fréchet-Urysohn; strongly Fréchet; productively Fréchet; hemicompact; locally compact; dual group.

©2015 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.