http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 47, 2016

Pages 273–278

http://topology.nipissingu.ca/tp/

# Remarks on metrizability of dual groups

by

Frédéric Mynard and Mikhail Tkachenko

Electronically published on October 21, 2015



**Topology Proceedings**

| | |
|---|---|
| **Web:** | http://topology.auburn.edu/tp/ |
| **Mail:** | Topology Proceedings |
| | Department of Mathematics & Statistics |
| | Auburn University, Alabama 36849, USA |
| **E-mail:** | topolog@auburn.edu |
| **ISSN:** | 0146-4124 |

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

**TOPOLOGY PROCEEDINGS**
Volume 47 (2016)
Pages 273-278

E-Published on October 21, 2015

# REMARKS ON METRIZABILITY OF DUAL GROUPS

FRÉDÉRIC MYNARD AND MIKHAIL TKACHENKO

Abstract. We examine sufficient conditions for the dual of a topological group to be metrizable and locally compact, improving on results of [4].

## 1. Introduction and preliminaries

If $G$ is an abelian topological group, its dual $\widehat{G}$ is the set of continuous group homomorphisms into the one-dimensional torus $\mathbb{T}$, endowed with the compact-open topology.

Two families $\mathcal{A}$ and $\mathcal{B}$ of subsets of a set $X$ are said to *mesh,* in symbols $\mathcal{A}\#\mathcal{B}$, if $A \cap B \neq \varnothing$ whenever $A \in \mathcal{A}$ and $B \in \mathcal{B}$. Thus, if the set $\mathbb{F}X$ of filters on a set $X$ is ordered by inclusion, two filters $\mathcal{F}$ and $\mathcal{G}$ admit a supremum $\mathcal{F} \vee \mathcal{G}$ if and only if $\mathcal{F}\#\mathcal{G}$. We do not distinguish between a sequence $(x_n)_{n\in\omega}$ and the filter generated by its tails. In particular, if $\mathcal{H}$ is a family of subsets of $X$ and $(x_n)_{n\in\omega}$ is a sequence on $X$, the notation $\mathcal{H}\#(x_n)_{n\in\omega}$ means that $H \cap \{x_n : n \geq k\} \neq \varnothing$ for every $H \in \mathcal{H}$ and $k \in \omega$.

Recall that a topological space $X$ is *sequential* if every sequentially closed subset is closed. The space is *Fréchet-Urysohn,* or simply *Fréchet,* if whenever $x \in X$ and $A \subset X$ with $x \in \operatorname{cl} A$, there is a sequence on $A$ that converges to $x$. It is *strongly Fréchet* if whenever $(A_n)_{n\in\omega}$ is a decreasing sequence of subsets of $X$ with $x \in \bigcap_{n\in\omega} \operatorname{cl} A_n$ there is $x_n \in A_n$ for each $n$ with $x_n \to_n x$, equivalently if

$$\forall \mathcal{H} \in \mathbb{F}_1 X,\ \operatorname{adh} \mathcal{H} \subseteq \operatorname{adh}_{\mathrm{Seq}} \mathcal{H},$$

2010 *Mathematics Subject Classification.* 54H11; 54A20; 54C35; 22A05.

*Key words and phrases.* Topological group; metrizability; Fréchet-Urysohn; strongly Fréchet; productively Fréchet; hemicompact; locally compact; dual group.