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THE TORSION BOHR COMPACTIFICATION OF ABELIAN GROUPS

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Abstract. Let G be an abstract abelian group and $G^\natural$ be the underlying group G endowed with the *torsion Bohr topology*, i.e., the topology on G induced by the family $G^{\circledast}$ of *all* homomorphisms of G to the torsion subgroup of the circle group $\mathbb{T}$. The completion of $G^\natural$ is known as the *torsion Bohr compactification* of G and is denoted by $\mathfrak{b}G$. The main results of the article are as follows:

(1) The group $\mathfrak{b}\mathbb{Z}$ is topologically isomorphic to $\Delta_{\mathfrak{a}}$, the additive group of $\mathfrak{a}$-adic integers with $\mathfrak{a} = (2, 3, 4, 5, \dots)$, where $\mathbb{Z}$ is the group of integers. (2) If G is divisible, then $\mathfrak{b}G$ contains a closed subgroup topologically isomorphic to a power of the $\mathfrak{a}$-adic solenoid with $\mathfrak{a} = (2, 3, 4, 5, \dots)$ multiplied by a product of powers of p-adic integers, with prime p's. (3) The group G is divisible if and only if $\mathfrak{b}G$ is divisible. (4) If $\mathfrak{b}G$ is zero-dimensional, then the group G is reduced, i.e., the unique divisible subgroup of G is $\{0\}$. Furthermore, $\mathfrak{b}G$ is zero-dimensional if and only if $G^{\circledast}$ is torsion if and only if G is isomorphic to $\mathbb{Z}^n \oplus tor(G)$ for some integer $n \geq 0$, where $tor(G)$ is a bounded torsion group. (5) If H is a subgroup of G, then $\mathfrak{b}(G/H) \cong \mathfrak{b}G/\mathfrak{b}H$ and the same relation is valid for the Bohr compactification, i.e., $b(G/N) \cong bG/bH$.

1. Introduction

The *torsion Bohr topology* on abelian groups was defined and studied in [2]. It admits a simple description as follows. Let G be an abstract abelian group. The coarsest topological group topology on G that makes every homomorphism of G to the torsion subgroup of the circle group $\mathbb{T}$

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