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CHARACTERIZATIONS OF THE PSEUDO-ARC

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Abstract. The pseudo-arc has many interesting properties and is still the subject of many significant questions. It has several characterizations. Some of these are well known while others are less well known. We review many of these characterizations. We also consider several possible additional characterizations as well as conditions which are known not to characterize the pseudo-arc.

1. Introduction

A *continuum* is a compact, connected metric space. A continuum X is *chainable* if, for every $\epsilon > 0$, there is an open cover $\mathcal{C} = \{C_1, C_2, \ldots, C_n\}$ of X such that $\text{diam}(C_i) < \epsilon$ for each $1 \leq i \leq n$ and $C_i \cap C_j \neq \emptyset$ if and only if $|i - j| \leq 1$. Nondegenerate chainable continua are also referred to as *arc-like* since they are homeomorphic to inverse limits of arcs and, for each $\epsilon > 0$, they admit a continuous surjection $f : X \to [0,1]$ with $\text{diam}(f^{-1}(t)) < \epsilon$ for each $0 \leq t \leq 1$.

A continuum is *indecomposable* if it is not the union of two proper subcontinua and *hereditarily indecomposable* if every subcontinuum of it is indecomposable.

A continuum X is *homogeneous* if, for each $x_1, x_2 \in X$, there is a homeomorphism $h : X \to X$ with $h(x_1) = x_2$.

2. First Characterizations

In 1922, B. Knaster [34] gave the first example of a nondegenerate hereditarily indecomposable continuum. He constructed it as the intersection of a nested sequence of strips, using what he termed the *method*

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