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by

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# TRAJECTORIES OF CHAOTIC INTERVAL MAPS

SHARAN GOPAL

Abstract. This paper proves the existence of an abundant number of sequences in $[0,1]$ that can occur as trajectories for chaotic interval maps. It is proved that given an allowed sequence in $[0,1]$ of certain kind, there always exists a chaotic interval map with this sequence as a trajectory.

## 1. Introduction

By a *dynamical system* $(X, f)$, we mean a topological space $X$ and a continuous self map $f$ on it. The *trajectory* of a point $x \in X$ is the sequence $(x, f(x), f^2(x), \ldots)$, where $f^n = f \circ f \circ f \ldots \circ f(n \text{ times})$ and the set $\{f^n(x) : n \in \mathbb{N}_0\}$ is called the *orbit* of $x$ ($\mathbb{N}_0$ is the set of non-negative integers and $f^0(x) = x$). If $f^n(x) = x$ for some $n \in \mathbb{N}$, then $x$ is called a *periodic point*. An *interval map* is a dynamical system, where the underlying topological space is $[0,1]$, i.e., a system of the form $([0,1], f)$.

**Definition 1.1** (See [3]). Let $(X, f)$ be a dynamical system, where $X$ is a metric space with metric $d$. $(X, f)$ is said to be *Devaney chaotic* if

(1) $f$ has sensitive dependence on initial conditions (i.e., there is an $r > 0$ such that for each point $x \in X$ and for each $\epsilon > 0$ there is a point $y \in X$ with $d(x, y) < \epsilon$ and a $k \geq 0$ such that $d(f^k(x), f^k(y)) \geq r$),

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*Key words and phrases.* Dynamical system; interval map; chaos.

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