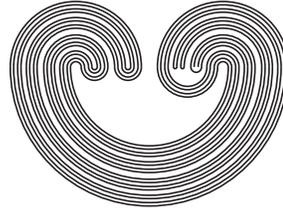


<http://topology.auburn.edu/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 48, 2016

Pages 233–249

---

<http://topology.nipissingu.ca/tp/>

A FINITE PRESENTATION FOR THE  
AUTOMORPHISM GROUP OF THE FIRST  
HOMOLOGY OF A NON-ORIENTABLE  
SURFACE OVER  $\mathbb{Z}_2$  PRESERVING THE MOD 2  
INTERSECTION FORM

by

RYOMA KOBAYASHI AND GENKI OMORI

Electronically published on September 17, 2015

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

## Topology Proceedings

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

**A FINITE PRESENTATION FOR THE AUTOMORPHISM  
GROUP OF THE FIRST HOMOLOGY OF A  
NON-ORIENTABLE SURFACE OVER  $\mathbb{Z}_2$  PRESERVING  
THE MOD 2 INTERSECTION FORM**

RYOMA KOBAYASHI AND GENKI OMORI

**ABSTRACT.** Let  $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$  be the group of automorphisms on the first homology group with  $\mathbb{Z}_2$  coefficients of a closed non-orientable surface  $N_g$  preserving the mod 2 intersection form. In this paper, we obtain a finite presentation for  $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$ . As an application we calculate the second homology group of  $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$ .

**1. INTRODUCTION**

For  $g \geq 1$  and  $n \geq 0$ , let  $N_{g,n}$  be a compact connected non-orientable surface of genus  $g$  with  $n$  boundary components (we denote  $N_{g,0}$  by  $N_g$ ) and a bilinear form  $\cdot \cdot : H_1(N_g; \mathbb{Z}_2) \times H_1(N_g; \mathbb{Z}_2) \rightarrow \mathbb{Z}_2$  the mod 2 intersection form on the first homology group  $H_1(N_g; \mathbb{Z}_2)$  of  $N_g$  with  $\mathbb{Z}_2$  coefficients. We represent  $N_g$  by a sphere with  $g$  crosscaps as in Figure 1; i.e., we regard  $N_g$  as a sphere with  $g$  boundary components and a Möbius band attached to each boundary component. We define  $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$  by the subgroup of the automorphism group  $\text{Aut } H_1(N_g; \mathbb{Z}_2)$  of  $H_1(N_g; \mathbb{Z}_2)$  preserving the mod 2 intersection form  $\cdot \cdot$ . Note that  $\text{Aut}(H_1(N_g; \mathbb{Z}_2), \cdot)$  is isomorphic to  $O(g, \mathbb{Z}_2) = \{A \in GL(g, \mathbb{Z}_2) \mid {}^tAA = E\}$  by taking the basis  $\{x_1, x_2, \dots, x_g\}$  for  $H_1(N_g; \mathbb{Z}_2)$ , where  $x_i$  is a homology class of a one-sided simple closed curve  $\mu_i$  in Figure 1 and  $E$  is an identity matrix

---

2010 *Mathematics Subject Classification.* 57M05, 57M07, 57M60.

*Key words and phrases.* group action on homology group, mapping class group, non-orientable surface.

The second author was supported by JSPS KAKENHI Grant number 15J10066.

©2015 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.