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by

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ADDING A CONVERGENT SEQUENCE

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Abstract. Let p be a non-isolated point in a space X. Suppose that no sequence $\{a_n : n < \omega\} \subseteq X \setminus \{p\}$ converges to p. We investigate in what circumstances can a cardinal-preserving forcing add a sequence $\{a_n : n < \omega\} \subseteq X \setminus \{p\}$ that converges to p.

1. Introduction

Let $\langle X, \tau \rangle$ be a topological space and let $\mathbb{P}$ be a notion of forcing. Let $\mathbf{V}$ be a ground model and let $\mathbf{V}^{\mathbb{P}}$ be the forcing extension of $\mathbf{V}$ by $\mathbb{P}$. We define in $\mathbf{V}^{\mathbb{P}}$ a topological space $\langle X, \tau^{\mathbb{P}} \rangle$ such that $\tau^{\mathbb{P}} = \{\bigcup S : S \subseteq \tau\}$; that is, $\tau^{\mathbb{P}}$ is the topology on X generated by τ in $\mathbf{V}^{\mathbb{P}}$. We observe that in general $\tau \subsetneqq \tau^{\mathbb{P}}$ because new open sets are added by $\mathbb{P}$. Also we note that τ serves as a base for $\tau^{\mathbb{P}}$.

Let p be a non-isolated point in a space $\langle X, \tau \rangle$. Suppose that in $\mathbf{V}$ no sequence $\{a_n : n < \omega\} \subseteq X \setminus \{p\}$ converges to p. We investigate in what circumstances can we add a sequence $\{a_n : n < \omega\} \subseteq X \setminus \{p\}$ which converges to p in the space $\langle X, \tau^{\mathbb{P}} \rangle$ by a cardinal-preserving forcing $\mathbb{P}$.

First let us illustrate an example where a forcing adds a convergent sequence.

Example 1.1. There exist a space $\langle X, \tau \rangle$, a point $p \in X$ and a forcing $\mathbb{P}$ with the countable chain condition (ccc) such that:

(1) in $\langle X, \tau \rangle$, no sequence $\{a_n : n < \omega\} \subseteq X \setminus \{p\}$ converges to p, and
(2) in $\langle X, \tau^{\mathbb{P}} \rangle$, there is a sequence $\{a_n : n < \omega\} \subseteq X \setminus \{p\}$ that converges to p.

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