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STRAIGHT HOMOTOPY INVARIANTS

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Abstract. Let X and Y be spaces and M be an abelian group. A homotopy invariant $f\colon [X,Y] \to M$ is called straight if there exists a homomorphism $F\colon L(X,Y) \to M$ such that $f([a]) = F(\langle a\rangle)$ for all $a \in C(X,Y)$. Here $\langle a\rangle\colon \langle X\rangle \to \langle Y\rangle$ is the homomorphism induced by a between the abelian groups freely generated by X and Y and $L(X,Y)$ is a certain group of "admissible" homomorphisms. We show that all straight invariants can be expressed through a "universal" straight invariant of homological nature.

1. Introduction

We define straight homotopy invariants of maps and give their characterization, which reduces them to the classical homology theory.

The group $L(X,Y)$. For a set X, let $\langle X\rangle$ be the (free) abelian group with the basis $X^\sharp \subseteq \langle X\rangle$ endowed with the bijection $X \to X^\sharp$, $x \mapsto {<}x{>}$. For sets X and Y, let $L(X,Y) \subseteq \mathrm{Hom}(\langle X\rangle, \langle Y\rangle)$ be the subgroup generated by the homomorphisms u such that $u(X^\sharp) \subseteq Y^\sharp \cup \{0\}$. (Elements of $L(X,Y)$ are the homomorphisms bounded with respect to the ℓ_1-norm.) A map $a\colon X \to Y$ induces the homomorphism $\langle a\rangle \in L(X,Y)$, $\langle a\rangle({<}x{>}) = {<}a(x){>}$.

Straight homotopy invariants. Let X and Y be spaces. Let $C(X,Y)$ be the set of continuous maps $X \to Y$ and $[X,Y]$ be the set of their homotopy classes. For $a \in C(X,Y)$, let $[a] \in [X,Y]$ be the homotopy class of a. Let M be an abelian group, and $f\colon [X,Y] \to M$ be a map (a homotopy invariant). The invariant f is called *straight* if there exists a homomorphism $F\colon L(X,Y) \to M$ such that $f([a]) = F(\langle a\rangle)$ for all $a \in C(X,Y)$.

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