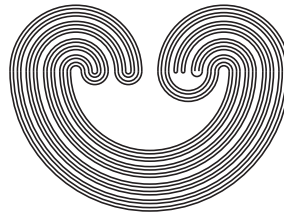


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ON GRAPH AND FINE TOPOLOGIES

by

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ON GRAPH AND FINE TOPOLOGIES

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ABSTRACT. Let X be a Tychonoff space and (Y, d) be a metric space. Let $C(X, Y)$ be the space of continuous functions from X to Y and τ_Γ, τ_w be the graph and fine topologies on $C(X, Y)$, respectively. Let (Y, d) contain a nontrivial path. We prove nontrivial generalizations of some known results concerning τ_Γ and τ_w on $C(X)$. For example the following are equivalent (1) $(C(X, Y), \tau_\Gamma) = (C(X, Y), \tau_w)$; (2) X is a *cb*-space. Some topological properties of $(C(X, Y), \tau_\Gamma)$ and $(C(X, Y), \tau_w)$ are studied too.

Let X be a topological space and (Y, d) be a metric space. We will suppose that X and Y are infinite. As usual let $C(X, Y)$ be the space of continuous functions from X to Y and $C(X)$ be the space of continuous real-valued functions.

As in [10] denote by $\tau_p, \tau_u, \tau_w, \tau_\Gamma$ the topology of pointwise convergence, the topology of uniform convergence, the fine topology and the graph topology on $C(X, Y)$, respectively. Of course $\tau_p \subseteq \tau_u \subseteq \tau_w \subseteq \tau_\Gamma$ on $C(X, Y)$.

Given a function $\epsilon : X \rightarrow (0, \infty)$ and $f \in C(X, Y)$, define

$$B(f, \epsilon) = \{g \in C(X, Y) : d(f(x), g(x)) < \epsilon(x) \text{ for all } x \in X\}.$$

Denote by $C^+(X)$ ($LSC^+(X)$) the set of all strictly positive real-valued continuous (lower semicontinuous) functions defined on X .

The fine topology τ_w on $C(X, Y)$ (also called *m*-topology [2]) has as a base all sets of the form $B(f, \epsilon)$, where ϵ runs over all elements from

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