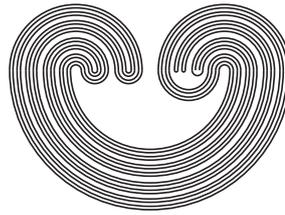


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## ON HOMOGENEITY AND THE H-CLOSED PROPERTY

by

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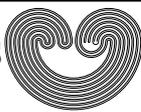
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## ON HOMOGENEITY AND THE H-CLOSED PROPERTY

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**ABSTRACT.** We establish several results concerning topological homogeneity and the weakening of compactness known as the H-closed property. First, it is shown that every Hausdorff space can be embedded in a homogeneous space that is the countable union of H-closed spaces. Second, it is shown that if  $X$  is an H-closed Urysohn homogeneous space then for every H-set set  $A \subseteq X$ ,  $x \in A$ , and  $y \notin A$ , there exists a homeomorphism  $h : X \rightarrow X$  such that  $h(y) \in A$  and  $h(x) \notin A$ . This is an extension of Motorov's result that every compact homogeneous space is 1.5-homogeneous. Third, we show that the cardinality bound  $2^{t(X)}$ , shown to hold for a compact homogeneous space  $X$  by De La Vega, does not hold in general for H-closed homogeneous spaces. Last, we show the Katětov H-closed extension  $\kappa X$  is never homogeneous if  $X$  is non-H-closed, and the remainder  $\sigma X \setminus X$  in the H-closed Fomin extension  $\sigma X$  is never power homogeneous if  $X$  is locally H-closed.

### 1. INTRODUCTION

A space  $X$  is *homogeneous* if for every  $x, y \in X$  there exists a homeomorphism  $h : X \rightarrow X$  such that  $h(x) = y$ .  $X$  is *power homogeneous* if there exists a cardinal  $\kappa$  such that  $X^\kappa$  is homogeneous. Many intriguing results have been obtained in the theory of compact homogeneous spaces; for example, De la Vega [11] showed that the cardinality of such a space  $X$  is at most  $2^{t(X)}$ , where  $t(X)$  is the tightness of  $X$ . Motorov showed that a compact homogeneous space has a stronger form of homogeneity known as  $1^{1/2}$ -homogeneity (see [2]). Many deep questions concerning these spaces are still open (see, for example Jan van Mill's survey in [17]).

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