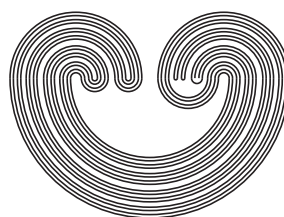


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OPEN-POINT TOPOLOGICAL GAMES AND PRODUCTIVITY OF DENSE-SEPARABLE PROPERTY

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OPEN-POINT TOPOLOGICAL GAMES AND PRODUCTIVITY OF DENSE-SEPARABLE PROPERTY

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ABSTRACT. In this note we study the open-point topological games in order to analyze the least upper bound for density of dense subsets of a topological space. This way we may also analyze the behavior of such cardinal invariants in taking products of spaces. Various related cardinal equalities and inequalities are given. As an application we take a look at Banach spaces with the property (CSP) which can be formulated by stating that each weak-star dense linear subspace of the dual is weak-star separable.

1. INTRODUCTION

Following [13] we call a topological space dense-separable if each of its dense subsets is separable (cf. [5]).

It is known (see [11]) that for compact spaces the dense-separability is equivalent to the countable π -weight. It follows that the property of being compact dense-separable space is preserved in countable products.

However, the problem whether a finite product of dense-separable topological spaces must be dense-separable appears to not have been settled. This problem is the starting point and the main motivation in this paper. The above problem is also closely related to some questions involving the structure of Banach spaces (see the discussion at the end, cf. [6, 14, 18]).

The preservation of a given property of topological spaces in finite products is a notoriously delicate problem, see [17]. In fact, often it is the case that a product of two spaces may fail to have that property even if both the spaces have the property, but the property of the product can be recovered by requiring one of the spaces to satisfy a stronger form of the given property.

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