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# MINIMALITY OF THE SEMIDIRECT PRODUCT

MICHAEL MEGRELISHVILI, LUIE POLEV, AND MENACHEM SHLOSSBERG

Abstract. A topological group is minimal if it does not admit a strictly coarser Hausdorff group topology. We provide a sufficient and necessary condition for the minimality of the semidirect product $G \leftthreetimes P$, where $G$ is a compact topological group and $P$ is a topological subgroup of $\mathrm{Aut}(G)$. We prove that $G \leftthreetimes P$ is minimal for every closed subgroup $P$ of $\mathrm{Aut}(G)$. In case $G$ is abelian, the same is true for every subgroup $P \subseteq \mathrm{Aut}(G)$. We show, in contrast, that there exist a compact two-step nilpotent group $G$ and a subgroup $P$ of $\mathrm{Aut}(G)$ such that $G \leftthreetimes P$ is not minimal. This answers a question of Dikranjan. Some of our results were inspired by a work of Gamarnik [12].

## 1. Introduction

A Hausdorff topological group $G$ is *minimal* ([10], [24]) if it does not admit a strictly coarser Hausdorff group topology or, equivalently, if every injective continuous group homomorphism $G \to P$ into a Hausdorff topological group is a topological group embedding. For information on minimal groups we refer to the surveys [6], [7], [8] and the book [9].

In [20] the two first-named authors study the minimality of the group $\mathrm{H}_+(X)$, where $X$ is a compact linearly ordered space and $\mathrm{H}_+(X)$ is the topological group of all order-preserving homeomorphisms of $X$. In general, $\mathrm{H}_+(X)$ need not be minimal. The first result in the present paper is Theorem 3.1, which shows that for a compact (partially) ordered space $X$ the compact-open topology on $\mathrm{H}_+(X, \leq)$ is minimal within the class

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