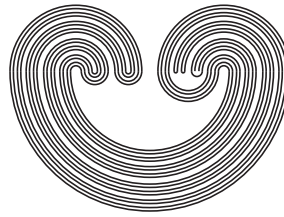


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MINIMALITY OF THE SEMIDIRECT PRODUCT

by

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MINIMALITY OF THE SEMIDIRECT PRODUCT

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ABSTRACT. A topological group is minimal if it does not admit a strictly coarser Hausdorff group topology. We provide a sufficient and necessary condition for the minimality of the semidirect product $G \rtimes P$, where G is a compact topological group and P is a topological subgroup of $\text{Aut}(G)$. We prove that $G \rtimes P$ is minimal for every closed subgroup P of $\text{Aut}(G)$. In case G is abelian, the same is true for every subgroup $P \subseteq \text{Aut}(G)$. We show, in contrast, that there exist a compact two-step nilpotent group G and a subgroup P of $\text{Aut}(G)$ such that $G \rtimes P$ is not minimal. This answers a question of Dikranjan. Some of our results were inspired by a work of Gamarnik [12].

1. INTRODUCTION

A Hausdorff topological group G is *minimal* ([10], [24]) if it does not admit a strictly coarser Hausdorff group topology or, equivalently, if every injective continuous group homomorphism $G \rightarrow P$ into a Hausdorff topological group is a topological group embedding. For information on minimal groups we refer to the surveys [6], [7], [8] and the book [9].

In [20] the two first-named authors study the minimality of the group $H_+(X)$, where X is a compact linearly ordered space and $H_+(X)$ is the topological group of all order-preserving homeomorphisms of X . In general, $H_+(X)$ need not be minimal. The first result in the present paper is Theorem 3.1, which shows that for a compact (partially) ordered space X the compact-open topology on $H_+(X, \leq)$ is minimal within the class

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