http://topology.auburn.edu/tp/



http://topology.nipissingu.ca/tp/

# Erratum to: On double spirals in Fibonacci-like unimodal inverse limit spaces

by

H. Bruin

Electronically published on November 3, 2016

## **Topology Proceedings**

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.



## ERRATUM TO: ON DOUBLE SPIRALS IN FIBONACCI-LIKE UNIMODAL INVERSE LIMIT SPACES

### H. BRUIN

Let  $\lim_{t \to \infty} ([c_2, c_1], T)$  be the core inverse limit space of a unimodal map T restricted to the core  $[c_2, c_1]$ . The purpose of the paper [3] was to create distinct rays C, C' inside  $\underline{\lim}([c_2, c_1], T)$  that converge to the same limit point, thus forming what was called a *double spiral*. However, the proof of Theorem 1 in [3] is false, because the backward itineraries associated to Cand C' cannot be simultaneously admissible. This follows from Lemma 1 below, to which I am indebted to Ana Anušić. In fact, Theorem 1 cannot be repaired, because it follows from a slight extension of [1, Proposition 1] that every subcontinuum of a unimodal inverse limit space contains a dense copy of  $\mathbb{R}$  having a single symbolic tail. Double spirals fail this property, regardless of the inverse limit space. The claim that the converse of Brucks & Diamond's result (namely that points with the same symbolic tail belong the same arc-component, [2, Lemma 2.8]) is false still stands. Indeed, some unimodal inverse limit spaces contain copies of  $\mathbb{R}$ converging on either side to a point, see e.g. the bar F in the arc + ray continuum of Example 3 in [1]. The resulting arc has three symbolic tails.

**Lemma 1.** Let  $a_1 < a_2 < a_3 < a_4 < a_5$  be positive integers. Then for any tent map,  $\varprojlim([c_2, c_1], T)$  does not contain simultaneously arcs A and A' with folding patterns  $a_1, a_3, a_5$  and  $a_1, a_2, a_1, a_3, a_4, a_3, a_5$ .

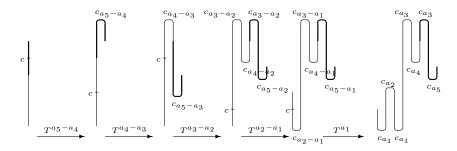
*Proof.* Let A' be an arc with folding pattern  $a_1, a_2, a_1, a_3, a_4, a_3, a_5$ . Then the projections  $\pi_{-a_5}, \ldots, \pi_{-a_1}$  of A' and maps  $T^{a_5-a_4}, \ldots, T^{a_2-a_1}$  are as

255

<sup>2010</sup> Mathematics Subject Classification. 37B45, 37E05, 54H20. ©2016 Topology Proceedings.

H. BRUIN

in the below figure, so  $c_{a_1} < c_{a_2}, c_{a_4}, c_{a_5} < c_{a_3}$  or in the reverse order. In bold, the arc is drawn that should correspond to A with folding pattern  $a_1, a_3, a_5$ . However, because  $c_{a_4} > c_{a_1}$ , it is impossible to extend A and reach a fold with level  $a_1$  before visiting a fold with level  $a_4$ .



#### References

- K. Brucks, H. Bruin, Subcontinua of inverse limit spaces of unimodal maps, Fund. Math. 160 (1999) 219–246.
- [2] K. Brucks, B. Diamond, A symbolic representation of inverse limit spaces for a class of unimodal maps, in Continuum Theory and Dynamical Systems, Lect. Notes in Pure and Appl. Math. 149 (1995) 207–226.
- [3] H. Bruin, On double spirals in Fibonacci-like unimodal inverse limit spaces, Topology Proc. 43 (2014), 83–97.

Faculty of Mathematics, University of Vienna, Oskar Morgensternplatz 1, 1090 Vienna, Austria

*E-mail address:* henk.bruin@univie.ac.at

256