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REVERSIBLE SPACES AND PRODUCTS

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Abstract. We demonstrate that there are connected reversible spaces such that their product is not reversible. This is an answer to a question posed by Chatyrko and Hattori.

1. Introduction

In [3] Rajagopalan and Wilansky introduced and studied reversible spaces, i.e. spaces X such that each continuous bijection $f\colon X \to X$ of X onto itself is a homeomorphism. Simple examples of reversible spaces are Hausdorff compacta and discrete spaces D_τ of cardinality $\tau \geq 1$. Other examples of reversible spaces are locally Euclidean spaces (in particular, n-dimensional manifolds without boundary). Even some manifolds with a non-empty boundary can be reversible, for example, the half-open interal $[0,1)$. Recall that there are many non-reversible manifolds (see [2]).

In [1] Chatyrko and Hattori observed that any product of topological spaces is non-reversible whenever at least one of the spaces is non-reversible. However, a product of two reversible spaces can be reversible and non-reversible as well. For example, $\mathbb{R} \times \mathbb{R}$ is reversible, while $D_{\aleph_0} \times [0,1)$ is not reversible. They asked if the topological product of two connected reversible spaces is reversible.

In this paper we suggest two connected reversible spaces such that their product is not reversible. Moreover, one of theses spaces is a 2-dimensional non-compact connected manifold in $\mathbb{R}^3$ without boundary, and the other is the closed interval $[0,1]$.

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