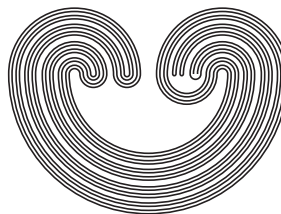


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## PRESERVATION OF COUNTABLE COMPACTNESS AND PSEUDOCOMPACTNESS BY FORCING

by

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## PRESERVATION OF COUNTABLE COMPACTNESS AND PSEUDOCOMPACTNESS BY FORCING

AKIRA IWASA

**ABSTRACT.** We study conditions under which countable compactness and pseudocompactness are preserved by forcing that satisfies the countable covering property.

### 1. INTRODUCTION

Let  $\mathbf{V}$  be a ground model and let  $\mathbb{P}$  be a forcing notion. Let  $\mathbf{V}^{\mathbb{P}}$  denote the forcing extension of  $\mathbf{V}$  by  $\mathbb{P}$ . For a topological space  $\langle X, \tau \rangle$  in  $\mathbf{V}$ , we define a topological space  $\langle X, \tau^{\mathbb{P}} \rangle$  in  $\mathbf{V}^{\mathbb{P}}$  such that  $\tau^{\mathbb{P}}$  is the topology generated by  $\tau$  in  $\mathbf{V}^{\mathbb{P}}$ . Note that we have in general  $\tau \subsetneq \tau^{\mathbb{P}}$  because new open sets are introduced by  $\mathbb{P}$ . Also note that by definition  $\tau$  is a base for  $\tau^{\mathbb{P}}$ .

We say that a forcing  $\mathbb{P}$  preserves a topological property  $\varphi$  if, whenever  $\langle X, \tau \rangle$  satisfies  $\varphi$ ,  $\langle X, \tau^{\mathbb{P}} \rangle$  satisfies  $\varphi$  as well. (In other words, we say that  $\mathbb{P}$  preserves  $\varphi$  if, whenever  $X$  satisfies  $\varphi$  in  $\mathbf{V}$ ,  $X$  satisfies  $\varphi$  in  $\mathbf{V}^{\mathbb{P}}$ .) Note that Hausdorffness, regularity and Tychonoffness are preserved by any forcing ([3] Lemma 22).

The following result is important for our study and it was noticed independently by several people (see, for example, [8, Lemma 7] and [1, Proposition 5.5]).

**Theorem 1.1.** *For a compact Hausdorff space  $X$ , the following are equivalent:*

- (1) *The compactness of  $X$  is preserved by any forcing.*
- (2) *The compactness of  $X$  is preserved by adjoining a Cohen real.*
- (3)  *$X$  is scattered.*

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