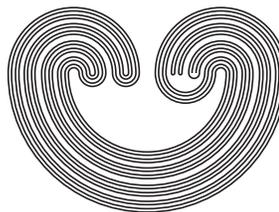


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PRESERVATION OF COUNTABLE COMPACTNESS AND PSEUDOCOMPACTNESS BY FORCING

by

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PRESERVATION OF COUNTABLE COMPACTNESS AND PSEUDOCOMPACTNESS BY FORCING

AKIRA IWASA

ABSTRACT. We study conditions under which countable compactness and pseudocompactness are preserved by forcing that satisfies the countable covering property.

1. INTRODUCTION

Let \mathbf{V} be a ground model and let \mathbb{P} be a forcing notion. Let $\mathbf{V}^{\mathbb{P}}$ denote the forcing extension of \mathbf{V} by \mathbb{P} . For a topological space $\langle X, \tau \rangle$ in \mathbf{V} , we define a topological space $\langle X, \tau^{\mathbb{P}} \rangle$ in $\mathbf{V}^{\mathbb{P}}$ such that $\tau^{\mathbb{P}}$ is the topology generated by τ in $\mathbf{V}^{\mathbb{P}}$. Note that we have in general $\tau \subsetneq \tau^{\mathbb{P}}$ because new open sets are introduced by \mathbb{P} . Also note that by definition τ is a base for $\tau^{\mathbb{P}}$.

We say that a forcing \mathbb{P} preserves a topological property φ if, whenever $\langle X, \tau \rangle$ satisfies φ , $\langle X, \tau^{\mathbb{P}} \rangle$ satisfies φ as well. (In other words, we say that \mathbb{P} preserves φ if, whenever X satisfies φ in \mathbf{V} , X satisfies φ in $\mathbf{V}^{\mathbb{P}}$.) Note that Hausdorffness, regularity and Tychonoffness are preserved by any forcing ([3] Lemma 22).

The following result is important for our study and it was noticed independently by several people (see, for example, [8, Lemma 7] and [1, Proposition 5.5]).

Theorem 1.1. *For a compact Hausdorff space X , the following are equivalent:*

- (1) *The compactness of X is preserved by any forcing.*
- (2) *The compactness of X is preserved by adjoining a Cohen real.*
- (3) *X is scattered.*

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