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by

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# INVERSE LIMITS OF ITERATES OF SET-VALUED FUNCTIONS

JAMES P. KELLY

Abstract. We present a function $F\colon [0,1] \to C([0,1])$ that is upper semi-continuous, and we show that if $n, m \in \mathbb{N}$ with $n \neq m$, then $\varprojlim F^n$ and $\varprojlim F^m$ are not homeomorphic. This answers a question posed by Matevž Črepnjak (2015). Additionally, we compare $F$ to two other functions: a continuous function $g\colon [0,1] \to [0,1]$ and an upper semi-continuous function $H\colon [0,1] \to 2^{[0,1]}$. We apply known results to state that $\varprojlim F$, $\varprojlim g$, and $\varprojlim H$ are all homeomorphic. We show, however, that the inverse limits of iterates of these functions are not homeomorphic to one another.

## 1. Introduction

The study of inverse limits of upper semi-continuous, set-valued functions is introduced by Williams S. Mahavier in [7] and further developed by W. T. Ingram and Mahavier in [4]. In these foundational papers, the authors demonstrate that many of the well-known properties which hold for inverse limits of continuous, single-valued functions do not always hold in the more general context of set-valued functions.

One such property is known as the subsequence property. This property implies that for a continuous function $f$ on a compact Hausdorff space $X$, the inverse limit of $f$ is homeomorphic to the inverse limit of $f^n$ for every natural number $n$. Ingram and Mahavier give two examples illustrating that the subsequence property does not always hold for upper semi-continuous, set-valued functions [4, Examples 3 & 4]. In [3, Problem 6.51], Ingram asks if there exists an upper semi-continuous, set-valued

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