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Orderability of Products

by

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ORDERABILITY OF PRODUCTS

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Abstract. We prove that for non-discrete spaces X and Y,

(1) if the product space $X \times Y$ is suborderable, then both X and Y are hereditarily paracompact and there is a unique regular infinite cardinal κ such that for every $z \in X \cup Y$, the cofinality from left (right) of z is either $0, 1$ or κ;
(2) if X and Y are subspaces of an ordinal, then the converse implication of (1) is also true.

1. Introduction

Recently, a kind of orderability of X^2 is known to be related to selection theory; see [5] and [3]. In this paper, we see the results in the abstract.

Spaces mean regular topological spaces. Let $<$ be a linear order on a set X. The usual order topology is denoted by $\lambda(<)$, that is, the topology generated by

$$\{(a, \rightarrow) : a \in X\} \cup \{(\leftarrow, b) : b \in X\}$$

as a subbase, where $(a, \rightarrow) = \{x \in X : a < x\}$, $(a, b) = \{x \in X : a < x < b\}$, etc. If necessary, we write $<_X$ and $(a, b)_X$ instead of $<$ and (a, b), respectively. A *linearly ordered topological space (LOTS)* X means the triple $\langle X, <, \lambda(<)\rangle$. As usual, we consider an ordinal α as the set of smaller ordinals and as a LOTS with the order $\in$ (we identify it with $<$). Similarly, a *generalized ordered space (GO-space)* means the triple $\langle X, <, \tau\rangle$ where τ is a topology on X with $\lambda(<) \subset \tau$ which has a base consisting of convex sets, where a subset A is *convex* if $(a, b) \subset A$ whenever $a, b \in A$ with $a < b$.

A topological space $\langle X, \tau\rangle$, where τ is a topology on X, is said to be *orderable* if $\tau = \lambda(<)$ for some linear order $<$ on X. Also, a topological

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