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KAKIMIZU COMPLEXES OF SURFACES AND 3-MANIFOLDS

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Abstract. The Kakimizu complex is usually defined in the context of knots, where it is known to be quasi-Euclidean. We here generalize the definition of the Kakimizu complex to surfaces and 3-manifolds (with or without boundary). Interestingly, in the setting of surfaces, the complexes and the techniques turn out to replicate those used to study the Torelli group, i.e., the "nonlinear" subgroup of the mapping class group. Our main results are that the Kakimizu complexes of a surface are contractible and that they need not be quasi-Euclidean. It follows that there exist (product) 3-manifolds whose Kakimizu complexes are not quasi-Euclidean.

The existence of Seifert's algorithm, discovered by Herbert Seifert, proves, among other things, that every knot admits a Seifert surface; i.e., for every knot K, there is a compact orientable surface whose boundary is K. It is worth noting that the existence of a Seifert surface for a knot K also follows from the existence of submanifolds representing homology classes of manifolds or pairs of submanifolds, in this case the pair $(K, \mathbb{S}^3)$. This point of view proves useful in generalizing our understanding of Seifert surfaces to other classes of surfaces in 3-manifolds.

Adding a trivial handle to a Seifert surface produces an isotopically distinct surface. Adding additional handles produces infinitely many isotopically distinct surfaces. These are not the multitudes of surfaces of primary interest here. The multitudes of surfaces of primary interest here are, for example, the infinite collection of Seifert surfaces produced by Julian R. Eisner [4]. Eisner realized that "spinning" a Seifert surface around the decomposing annulus of a connected sum of two non-fibered

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