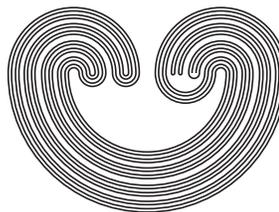


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## ROTATION NUMBERS OF ELEMENTS IN THOMPSON'S GROUP $T$

by

JEFFREY DILLER AND JAN-LI LIN

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## ROTATION NUMBERS OF ELEMENTS IN THOMPSON'S GROUP $\mathbf{T}$

JEFFREY DILLER AND JAN-LI LIN

**ABSTRACT.** We give a simple combinatorial proof that the rotation number for each element in Thompson's group  $\mathbf{T}$  is rational.

### 1. INTRODUCTION

In 1965, Richard Thompson defined three groups which have furnished counterexamples to various conjectures in group theory. One of these  $\mathbf{T}$ , which will be the subject of this article, is the group of dyadic circle homeomorphisms  $f : \mathbb{S}^1 \rightarrow \mathbb{S}^1$ . That is, if one takes  $\mathbb{S}^1$  to be the interval  $[0, 1]$  with endpoints identified, then

- $f$  preserves the set of dyadic rational numbers (i.e., numbers of the form  $p \cdot 2^q$ ,  $p, q \in \mathbb{Z}$ ),
- $f$  is linear except at a finite number of dyadic rational points,
- on each interval such that  $f$  is linear, the derivative (i.e., slope) is a power of 2.

For more information on Thompson's groups, see [5], [9], [10]. (Notice that the group  $\mathbf{T}$  is denoted by  $G$  in [9] and [10].)

The *rotation number* [11] of an orientation preserving circle homeomorphism  $f : \mathbb{S}^1 \rightarrow \mathbb{S}^1$  is the quantity

$$\rho(f) = \lim_{n \rightarrow \infty} \frac{\tilde{f}^n(x)}{n} \pmod{\mathbb{Z}},$$

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