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ROTATION NUMBERS OF ELEMENTS IN THOMPSON'S GROUP T

JEFFREY DILLER AND JAN-LI LIN

Abstract. We give a simple combinatorial proof that the rotation number for each element in Thompson's group **T** is rational.

1. Introduction

In 1965, Richard Thompson defined three groups which have furnished counterexamples to various conjectures in group theory. One of these **T**, which will be the subject of this article, is the group of dyadic circle homeomorphisms $f : \mathbb{S}^1 \to \mathbb{S}^1$. That is, if one takes $\mathbb{S}^1$ to be the interval $[0, 1]$ with endpoints identified, then

- f preserves the set of dyadic rational numbers (i.e., numbers of the form $p \cdot 2^q$, $p, q \in \mathbb{Z}$),
- f is linear except at a finite number of dyadic rational points,
- on each interval such that f is linear, the derivative (i.e., slope) is a power of 2.

For more information on Thompson's groups, see [5], [9], [10]. (Notice that the group **T** is denoted by G in [9] and [10].)

The *rotation number* [11] of an orientation preserving circle homeomorphism $f : \mathbb{S}^1 \to \mathbb{S}^1$ is the quantity

$$\rho(f) = \lim_{n\to\infty} \frac{\widetilde{f}^n(x)}{n} \mod \mathbb{Z},$$

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