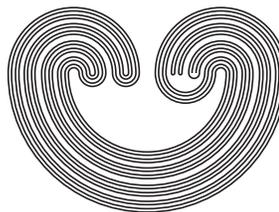


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## AN INTEGRAL WEIGHT REALIZATION THEOREM FOR SUBSET CURRENTS ON FREE GROUPS

by

ILYA KAPOVICH

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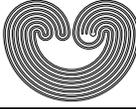
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## AN INTEGRAL WEIGHT REALIZATION THEOREM FOR SUBSET CURRENTS ON FREE GROUPS

ILYA KAPOVICH

**ABSTRACT.** We prove that if  $N \geq 2$  and  $\alpha : F_N \rightarrow \pi_1(\Gamma)$  is a marking on  $F_N$ , then, for any integer  $r \geq 2$  and any  $F_N$ -invariant collection of non-negative integral “weights” associated to all subtrees  $K$  of  $\tilde{\Gamma}$  of radius  $\leq r$  satisfying some natural “switch” conditions, there exists a finite cyclically reduced folded  $\Gamma$ -graph  $\Delta$  realizing these weights as numbers of “occurrences” of  $K$  in  $\Delta$ . As an application, we give a new, direct, and explicit proof of one of the main results of our paper with Tatiana Nagnibeda (*Subset currents on free groups*, *Geom. Dedicata* **166** (2013), 307–348) stating that, for any  $N \geq 2$ , the set  $\mathcal{SCurr}_r(F_N)$  of all rational subset currents is dense in the space  $\mathcal{SCurr}(F_N)$  of subset currents on  $F_N$ . (The proof given in the above-cited paper was indirect and omitted significant details. The proof given here is complete and, we hope, more accessible to the  $Out(F_N)$  community.)

We also answer one of the questions (Problem 10.11) posed in the above-mentioned paper. Thus, we prove that if a nonzero  $\mu \in \mathcal{SCurr}(F_N)$  has all weights with respect to some marking being integers, then  $\mu$  is the sum of finitely many “counting” currents corresponding to nontrivial finitely generated subgroups of  $F_N$ .

### 1. INTRODUCTION

The main purpose of this paper is to give a proof of Theorem B below (originally established in [24] via an indirect argument) which is self-contained, direct, explicit, and can be relatively easily understood by the

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