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## A NEW ASPECT OF SPACES OF COUNTABLE PSEUDOCHARACTER

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## A NEW ASPECT OF SPACES OF COUNTABLE PSEUDOCHARACTER

VLADIMIR V. TKACHUK

**ABSTRACT.** We introduce and study the spaces with  $\kappa$ -representative families of pseudo-networks for any infinite cardinal  $\kappa$ . We show that the respective classes are invariant under arbitrary subspaces, countable products, and lifted by condensations. Furthermore, the class of spaces with  $\kappa$ -representative families of pseudo-networks is preserved by  $\sigma$ -products. It turns out that every space of countable pseudocharacter has a representative family of pseudo-networks. If  $X$  is a subspace of an ordinal, then  $X$  has an  $\omega$ -representative family of pseudo-networks. We also establish that if a space  $X$  has a representative family of countable pseudo-networks, then  $L(X) \cdot \psi(X) \leq \omega$  implies that  $|X| \leq \mathfrak{c}$ . This fact is new for monotonically monolithic spaces; in addition, it generalizes the respective results for spaces of countable tightness, monotonically normal spaces, and spaces of countable Hausdorff pseudocharacter.

### 1. INTRODUCTION

If  $X$  is a topological space, and  $A \subset X$ , then the set  $\overline{A}$  can be much larger than  $A$  so any method of analyzing the properties of  $\overline{A}$  could be useful for understanding the behavior of the topology of  $X$ . It is probable that A. V. Arhangel'skiĭ had this idea in mind when he discovered the class of monolithic spaces (see [3]). Recall that, for an infinite cardinal  $\kappa$ , a space  $X$  is called  $\kappa$ -monolithic if  $nw(\overline{A}) \leq \kappa$  for every set  $A \subset X$  with  $|A| \leq \kappa$ . The space  $X$  is *monolithic* if it is  $\kappa$ -monolithic for any infinite cardinal  $\kappa$ .

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