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NONCONNECTED INVERSE LIMITS

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Abstract. In this paper we give an example of an inverse limit sequence on $[0,1]$ with a single upper semi-continuous set-valued bonding function f such that $G(f^n)$ is an arc for each positive integer n, but the inverse limit is not connected. This answers a question posed by W. T. Ingram.

1. Introduction

In [1] Iztok Banič and Judy Kennedy pose a question: If $f:[0,1]\to 2^{[0,1]}$ is an upper semi-continuous function such that $G(f)$ is an arc and $G(f^n)$ is connected for each positive integer n, is $\varprojlim f$ connected? In [2] W. T. Ingram answers their question in the negative (see Example 1) and asked whether f produces a connected inverse limit in case $G(f^n)$ is an arc for each positive integer n. In this paper we give a negative answer to this question.

2. Definitions and Notation

A *continuum* is a non-empty compact connected metric space. If X is a continuum, $2^X=\{A\subseteq X : A \text{ is non-empty closed in } X\}$ denotes the *hyperspace* of X. If X and Y are continua, a function $f:X\to 2^Y$ is said to be *upper semi-continuous* if for every $x_0\in X$ and every open subset U of Y such that $f(x_0)\subset U$, the set $\{x\in X : f(x)\subset U\}$ is an open subset of X. The *graph* of the function $f:X\to 2^Y$ is $G(f)=\{(x,y): y\in f(x)\}$, and for a subset A of X, we define $f(A)=\{y\in Y : y\in f(x) \text{ for some } x\in A\}$. If $f:X\to 2^X$, then we denote the composition $f\circ f$ by f^2 and, for any integer $n>2$, $f^n=f^{n-1}\circ f$.

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