http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 50, 2017

Pages 319–333

http://topology.nipissingu.ca/tp/

Continuous Injections Between the Products of Two Connected Nowhere Real Linearly Ordered Spaces

by

Tetsuya Ishiu

Electronically published on March 15, 2017

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 50 (2017)
Pages 319-333

E-Published on March 13, 2017

CONTINUOUS INJECTIONS BETWEEN THE PRODUCTS OF TWO CONNECTED NOWHERE REAL LINEARLY ORDERED SPACES

TETSUYA ISHIU

Abstract. We shall show that if K_0, K_1, L_0, and L_1 are nowhere real connected linearly ordered topological spaces and $f : K_0 \times K_1 \to L_0 \times L_1$ is a continuous injective function, then f is coordinate-wise.

1. Introduction

Let $f : X_0 \times X_1 \to Y_0 \times Y_1$ be a function. We say that f is *coordinate-wise* if and only if there exist $i < 2$, $g_0 : X_i \to Y_0$, and $g_1 : X_{1-i} \to Y_1$ such that for every $\langle x_0, x_1\rangle \in X_0 \times X_1$, $f(x_0, x_1) = \langle g_0(x_i), g_1(x_{1-i})\rangle$.

Many homeomorphisms from $\mathbb{R}^2$ onto $\mathbb{R}^2$ are not coordinate-wise. For example, $f(x, y) = \langle x - y, x + y\rangle$.

However, K. Eda and R. Kamijo proved the following theorem that this is not necessarily the case when we replace $\mathbb{R}$ by other connected linearly ordered spaces.

Theorem 1.1 (Eda and Kamijo [1]). *Let K be a connected linearly ordered space such that, for a dense set of $x \in K$, either $\mathrm{cf}(x)$ or $\mathrm{ci}(x)$ is uncountable. Here, $\mathrm{cf}(x)$ denotes the cofinality of x and $\mathrm{ci}(x)$ the coinitiality of x. Then for every $n < \omega$, every homeomorphism $f : K^n \to K^n$ is coordinate-wise.*

Eda and Kamijo asked if it can be extended to, for example, the cut-completion of an Aronszajn line. In this article, we shall prove the following theorem that answers this question positively with some other improvements for the case $n = 2$.

2010 *Mathematics Subject Classification.* Primary 03E75, 54B10, 54F05.
Key words and phrases. linearly ordered topological spaces, product spaces.
©2017 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.