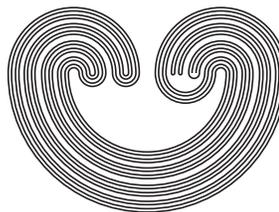


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A GEOMETRIC PROOF OF THE STRUCTURE THEOREM FOR CYCLIC SPLITTINGS OF FREE GROUPS

by

CHRISTOPHER H. CASHEN

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Auburn University, Alabama 36849, USA

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A GEOMETRIC PROOF OF THE STRUCTURE THEOREM FOR CYCLIC SPLITTINGS OF FREE GROUPS

CHRISTOPHER H. CASHEN

ABSTRACT. We give a geometric proof of a well-known theorem that describes splittings of a free group as an amalgamated product or HNN extension over the integers. The argument generalizes to give a similar description of splittings of a virtually free group over a virtually cyclic group.

1. INTRODUCTION

This paper describes one-edge splittings of free groups over (infinite) cyclic subgroups. Conversely, it describes when two free groups can be amalgamated along a cyclic subgroup to form a free group, or when an HNN-extension of a free group along a cyclic subgroup is free.

Theorem 1.1 (Shenitzer [17], Stallings [19], Swarup [20]). *Let A and B be finitely generated free groups, and let C be a cyclic group.*

- $G = A *_C B$ is free if and only if one of the injections of C into A and B maps C onto a free factor of the vertex group.
- $G = A *_C$ is free if and only if, up to A -conjugation, the edge injections map C into independent free factors of A , and one of them is onto its factor.

This theorem is well known. The amalgamated product case is a theorem of Abe Shenitzer [17]. The HNN case follows from a theorem of

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