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# A GEOMETRIC PROOF OF THE STRUCTURE THEOREM FOR CYCLIC SPLITTINGS OF FREE GROUPS

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Abstract. We give a geometric proof of a well-known theorem that describes splittings of a free group as an amalgamated product or HNN extension over the integers. The argument generalizes to give a similar description of splittings of a virtually free group over a virtually cyclic group.

## 1. Introduction

This paper describes one-edge splittings of free groups over (infinite) cyclic subgroups. Conversely, it describes when two free groups can be amalgamated along a cyclic subgroup to form a free group, or when an HNN-extension of a free group along a cyclic subgroup is free.

**Theorem 1.1** (Shenitzer [17], Stallings [19], Swarup [20])**.** *Let $A$ and $B$ be finitely generated free groups, and let $C$ be a cyclic group.*

- *$G = A *_C B$ is free if and only if one of the injections of $C$ into $A$ and $B$ maps $C$ onto a free factor of the vertex group.*
- *$G = A*_C$ is free if and only if, up to $A$-conjugation, the edge injections map $C$ into independent free factors of $A$, and one of them is onto its factor.*

This theorem is well known. The amalgamated product case is a theorem of Abe Shenitzer [17]. The HNN case follows from a theorem of

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