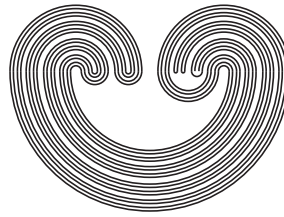


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CLOPEN ULTRAFILTERS OF ω AND THE CARDINALITY OF THE STONE SPACE $S(\omega)$ IN **ZF**

by

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CLOPEN ULTRAFILTERS OF ω AND THE CARDINALITY OF THE STONE SPACE $S(\omega)$ IN ZF

KYRIAKOS KEREMEDIS

ABSTRACT. For $X \in \{\omega, \mathbb{R}\}$ let $\mathbf{2}^X$ denote the Tychonoff product of the discrete space $\mathbf{2} = \{0, 1\}$ and, Cl_X denote the set $\{O \cap D_X : O \text{ is a clopen subset of } \mathbf{2}^X\}$ where, $D_X = \{d_n : n \in \omega\}$ is a dense subset of $\mathbf{2}^X$.

We show:

(i) ω has a free ultrafilter iff every Cl_ω - filter extends to an ultrafilter of D_ω iff every Cl_ω - ultrafilter extends to an ultrafilter of D_ω .

(ii) Every filter of ω extends to an ultrafilter of ω iff every $Cl_{\mathbb{R}}$ - filter extends to an ultrafilter of $D_{\mathbb{R}}$ iff every $Cl_{\mathbb{R}}$ - filter extends to a $Cl_{\mathbb{R}}$ - ultrafilter.

(iv) $\mathbf{2}^{\mathbb{R}}$ is the continuous image of $S(\omega)$ iff every $Cl_{\mathbb{R}}$ - ultrafilter extends to an ultrafilter of $D_{\mathbb{R}}$.

(v) If the Stone space $S(\omega)$ is countably compact then every family $\mathcal{A} = \{A_i, B_i\} \subseteq [\mathbb{R}]^\omega : i \in \omega\}$ has a choice set.

(vi) "Every filter of \mathbb{R} extends to an ultrafilter of \mathbb{R} " implies $|S(\omega)| = |\mathbb{R}|$, "every filter of ω extends to an ultrafilter of ω " implies $|2^{S(\omega)}| = |2^{\mathbb{R}}|$ and, " ω has a free ultrafilter" implies $|\mathbb{R}| \leq |S(\omega)|$.

1. NOTATION AND TERMINOLOGY

Let $\mathbf{X} = (X, T)$ be a topological space.

\mathbf{X} is said to be *compact* iff every open cover \mathcal{U} of \mathbf{X} has a finite subcover \mathcal{V} .

\mathbf{X} is said to be *countably compact* iff every countable open cover \mathcal{U} of \mathbf{X} has a finite subcover \mathcal{V} .

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