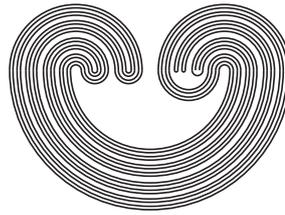


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TOPOLOGY PROCEEDINGS



Volume 51, 2018

Pages 133-141

<http://topology.nipissingu.ca/tp/>

DEGREES OF MAPS BETWEEN ISOTROPIC GRASSMANN MANIFOLDS

by

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Electronically published on August 26, 2017

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E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

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DEGREES OF MAPS BETWEEN ISOTROPIC GRASSMANN MANIFOLDS

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ABSTRACT. Let $\tilde{I}_{2n,k}$ denote the space of k -dimensional, oriented isotropic subspaces of \mathbb{R}^{2n} , called the oriented isotropic Grassmannian. Let $f: \tilde{I}_{2n,k} \rightarrow \tilde{I}_{2m,l}$ be a map between two oriented isotropic Grassmannians of the same dimension, where $k, l \geq 2$. We show that either $(n, k) = (m, l)$ or $\deg f = 0$. Let $\mathbb{R}\tilde{G}_{m,l}$ denote the oriented real Grassmann manifold. For $k, l \geq 2$ and $\dim \tilde{I}_{2n,k} = \dim \mathbb{R}\tilde{G}_{m,l}$, we also show that the degree of maps $g: \mathbb{R}\tilde{G}_{m,l} \rightarrow \tilde{I}_{2n,k}$ and $h: \tilde{I}_{2n,k} \rightarrow \mathbb{R}\tilde{G}_{m,l}$ must be zero.

1. INTRODUCTION

It has been proved in [5] that maps between two different oriented real Grassmann manifolds of the same dimension cannot have non-zero degree, provided the target space is not a sphere. A similar result is obtained for complex Grassmann manifolds in [4], when the map is a morphism of projective varieties. For arbitrary maps, this result has been verified for the complex Grassmann manifolds for many cases in [5] and [6].

In this paper we consider the analogous question for the space $\tilde{I}_{2n,k}$ of oriented k -dimensional isotropic subspaces of a symplectic vector space of dimension $2n$. The oriented isotropic Grassmannian was considered in [3] and its cohomology was computed with real coefficients. Their method involves identifying $\tilde{I}_{2n,k}$ as a homogeneous space $\tilde{I}_{2n,k} \simeq U(n)/(SO(k) \times U(n-k))$. One may similarly consider $I_{2n,k}$, the isotropic Grassmannian of k -dimensional isotropic subspaces of a symplectic $2n$ dimensional vector

2010 *Mathematics Subject Classification*. Primary 55M25, 14M17; Secondary 14M15, 57T15, 55R40.

Key words and phrases. Isotropic Grassmann Manifolds, Brouwer Degree, Characteristic Classes.

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