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David Maya, Patricia Pellicer-Covarrubias and Roberto Pichardo-Mendoza

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GENERAL PROPERTIES OF THE HYPERSPACE OF CONVERGENT SEQUENCES

DAVID MAYA, PATRICIA PELLICER-COVARRUBIAS,
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Abstract. Given a Hausdorff space X, the symbol $\mathcal{S}_c(X)$ denotes the topological space which results of endowing the set of all infinite convergent sequences in X with the Vietoris topology. This hyperspace was introduced in [5].

In this paper we present answers to some questions posed in that article, namely, we show that if X is either metrizable or second countable, then X is pathwise connected as long as $\mathcal{S}_c(X)$ is so, and we exhibit a dendroid X for which $\mathcal{S}_c(X)$ is not pathwise connected. Continuing with negative examples, we present a normal (resp. Fréchet-Urysohn) space whose hyperspace of converging sequences is not normal (resp. Fréchet-Urysohn).

By proving that the hypothesis X *is connected* implies that $\mathcal{S}_c(X)$ is connected we generalize one of the results from the article mentioned above. Moreover, it is proved here that the reverse implication holds whenever $\mathcal{S}_c(X) \neq \emptyset$ and similiar results are obtained when we replace *connected* with *locally connected*.

A section is included where the weight, the character and the density of $\mathcal{S}_c(X)$ are compared with the corresponding cardinal functions of X. Then we turn our attention to the study of the topological dimension of the hyperspace of convergent sequences of compact metrizable spaces. Finally, we characterize the continuous functions from $\mathcal{S}_c(X)$ to $\mathcal{S}_c(Y)$ which are inducible.

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