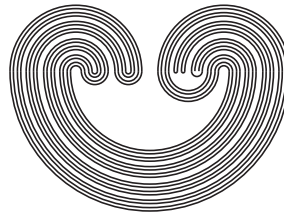


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by

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ON SPACES WITH RANK k -DIAGONALS OR ZEROSSET DIAGONALS

WEI-FENG XUAN AND WEI-XUE SHI

ABSTRACT. In this paper, we make some observations on spaces with rank k -diagonals or zerosset diagonals. In particular, we prove some cardinality inequalities on spaces with rank 2-diagonals or rank 3-diagonals. Moreover, we prove that if a space X has a zerosset diagonal and X^2 is star σ -compact then X is submetrizable.

1. INTRODUCTION

All spaces are assumed to be topological T_1 -spaces. All notation and terminology not explained in the paper is given in [6]. If A is a subset of a space X and \mathcal{U} is a family of subsets of X , then $\text{St}(A, \mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. We also put $\text{St}^0(A, \mathcal{U}) = A$ and for a natural number n , $\text{St}^{n+1}(A, \mathcal{U}) = \text{St}(\text{St}^n(A, \mathcal{U}), \mathcal{U})$. If $A = \{x\}$ for some $x \in X$, then we write $\text{St}^n(x, \mathcal{U})$ instead of $\text{St}^n(\{x\}, \mathcal{U})$.

A diagonal sequence of rank k on a space X , where $k \in \omega$, is a countable family $\{\mathcal{U}_n : n \in \omega\}$ of open covering of X such that $\{x\} = \bigcap\{\text{St}^k(x, \mathcal{U}_n) : n \in \omega\}$ for each $x \in X$. A space X has a rank k -diagonal, where $k \in \omega$, if there is a diagonal sequence $\{\mathcal{U}_n : n \in \omega\}$ on X of rank k . A space X has a strong rank k -diagonal, where $k \in \omega$, if there is a diagonal sequence $\{\mathcal{U}_n : n \in \omega\}$ on X such that $\{x\} = \bigcap\{\overline{\text{St}^k(x, \mathcal{U}_n)} : n \in \omega\}$ for each $x \in X$. The rank of the diagonal of X is defined as the greatest natural number k such that X has a rank k -diagonal, if such a number k exists. Note that every rank 3-diagonal implies regular G_δ -diagonal and every submetrizable space has a rank k -diagonal for each $k \in \omega$. For more details on rank k -diagonal, see [2].

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