http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 51, 2018

Pages 245–251

http://topology.nipissingu.ca/tp/

On spaces with rank k-diagonals or zeroset diagonals

by

Wei-Feng Xuan and Wei-Xue Shi

Electronically published on November 23, 2017

Topology Proceedings

Web: http://topology.auburn.edu/tp/
Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA
E-mail: topolog@auburn.edu
ISSN: (Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 51 (2018)
Pages 245-251

E-Published on November 23, 2017

ON SPACES WITH RANK k-DIAGONALS OR ZEROSET DIAGONALS

WEI-FENG XUAN AND WEI-XUE SHI

ABSTRACT. In this paper, we make some observations on spaces with rank k-diagonals or zeroset diagonals. In particular, we prove some cardinality inequalities on spaces with rank 2-diagonals or rank 3-diagonals. Moreover, we prove that if a space X has a zeroset diagonal and X^2 is star σ-compact then X is submetrizable.

1. INTRODUCTION

All spaces are assumed to be topological T_1-spaces. All notation and terminology not explained in the paper is given in [6]. If A is a subset of a space X and $\mathcal{U}$ is a family of subsets of X, then $\mathrm{St}(A,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. We also put $\mathrm{St}^0(A,\mathcal{U}) = A$ and for a natural number n, $\mathrm{St}^{n+1}(A,\mathcal{U}) = \mathrm{St}(\mathrm{St}^{\mathrm{n}}(A,\mathcal{U}),\mathcal{U})$. If $A = \{x\}$ for some $x \in X$, then we write $\mathrm{St}^{\mathrm{n}}(x,\mathcal{U})$ instead of $\mathrm{St}^{\mathrm{n}}(\{x\},\mathcal{U})$.

A diagonal sequence of rank k on a space X, where $k \in \omega$, is a countable family $\{\mathcal{U}_n : n \in \omega\}$ of open covering of X such that $\{x\} = \bigcap\{\mathrm{St}^{\mathrm{k}}(x,\mathcal{U}_n) : n \in \omega\}$ for each $x \in X$. A space X has a rank k-diagonal, where $k \in \omega$, if there is a diagonal sequence $\{\mathcal{U}_n : n \in \omega\}$ on X of rank k. A space X has a strong rank k-diagonal, where $k \in \omega$, if there is a diagonal sequence $\{\mathcal{U}_n : n \in \omega\}$ on X such that $\{x\} = \bigcap\{\overline{\mathrm{St}^{\mathrm{k}}(x,\mathcal{U}_n)} : n \in \omega\}$ for each $x \in X$. The rank of the diagonal of X is defined as the greatest natural number k such that X has a rank k-diagonal, if such a number k exists. Note that every rank 3-diagonal implies regular G_δ-diagonal and every submetrizable space has a rank k-diagonal for each $k \in \omega$. For more details on rank k-diagonal, see [2].

2010 *Mathematics Subject Classification.* Primary 54D20; Secondary 54E35.

Key words and phrases. Rank 2-diagonal, Rank 3-diagonal, Weakly Lindelöf, Normal, Cardinality, Zeroset diagonal, Star σ-compact.

The paper is supported by NSFC project 11626131 and 11271178.