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by

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QUANTALE-VALUED GENERALIZATIONS OF APPROACH SPACES: L-APPROACH SYSTEMS

GUNTHER JÄGER

Abstract. We define and study quantale-valued approach systems. We show that the resulting category is topological and study its relation to other categories of quantale-valued generalizations of approach spaces, such as the categories of quantale-valued approach spaces and of quantale-valued gauge spaces. We pay particular attention to the probabilistic case.

1. Introduction

The category of approach spaces, introduced in [13], is a common supercategory of the categories of metric and topological spaces. The theory of these spaces is far developed and has many applications as is demonstrated in e.g. [14, 15]. In simple terms one may say, that the theory of approach spaces is "metrical" in the sense that an approach space is often either defined by a *point-set distance function* or a suitable family of metrics (a so-called *gauge*) or by families of "local distances" (so-called *approach systems*). Therefore, the reservations that apply to metric spaces in terms of the precise knowledge of distances between elements apply also to approach spaces, and probabilistic generalizations seem natural. In [7], we introduced such a probabilistic generalization of approach spaces and suggested to even consider a further quantale-valued generalization. This was taken up in [10], who showed that such quantale-valued approach spaces fit nicely into the framework of monoidal topology [5]. In both the probabilistic case [7] and the quantale-valued case [10], the basic definition is in terms of a quantale-valued point-set distance function and also equivalent forms in terms of quantale-valued (ultra-)filter convergence are established.

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