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by

Daria Michalik

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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DARIA MICHALIK

Abstract. We prove that if X and Y are locally connected curves not being ANRs, then X and Y are homeomorphic if and only if $\mathrm{Cone}(X)$ and $\mathrm{Cone}(Y)$ are homeomorphic.

1. Introduction

Let X be a topological space. The *cone* of X is the quotient space defined by

$$\mathrm{Cone}(X) = X \times \mathbb{I} / (X \times \{1\}).$$

The *cylinder* of X is the Cartesian product $X \times \mathbb{I}$ and the *suspension* of X is the quotient space

$$\mathrm{Sus}(X) = X \times \mathbb{I} / (X \times \{0\},\ X \times \{1\}).$$

It is well known that cones of non-homeomorphic spaces can be homeomorphic, e.g., $\mathrm{Cone}(S^1)$ and $\mathrm{Cone}(\mathbb{I})$.

Example 1.1. Let A_i be a cone over i-point space and B_i be a suspension over i-point space. Then, for every $i \in \mathbb{N}$, $\mathrm{Cone}(A_i)$ and $\mathrm{Cone}(B_i)$ are homeomorphic, but A_i and B_i are not homeomorphic.

The main theorem of this note follows.

Theorem 1.2. *Let us assume that X and Y are locally connected curves not being ANRs. Then* $\mathrm{Cone}(X)$ *and* $\mathrm{Cone}(Y)$ *are homeomorphic if and only if X is homeomorphic to Y.*

Remark 1.3. By [3], if X and Y are locally connected curves, then the cylinder of X is homeomorphic to the cylinder of Y if and only if X is homeomorphic to Y.

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