http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 52, 2018

Pages 45–59

http://topology.nipissingu.ca/tp/

Dynamics of Non-Autonomous Discrete Dynamical Systems

by

Puneet Sharma and Manish Raghav

Electronically published on June 5, 2017

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 52 (2018)
Pages 45-59

E-Published on June 5, 2017

DYNAMICS OF NON-AUTONOMOUS DISCRETE DYNAMICAL SYSTEMS

PUNEET SHARMA AND MANISH RAGHAV

Abstract. We study the dynamics of a general non-autonomous dynamical system generated by a family of continuous self-maps on a compact space X. We derive necessary and sufficient conditions for the system to exhibit complex dynamical behavior. In the process we discuss properties like transitivity, weakly mixing, topologically mixing, minimality, sensitivity, topological entropy, and Li–Yorke chaoticity for the non-autonomous system. We also give examples to prove that the dynamical behavior of the non-autonomous system in general cannot be characterized in terms of the dynamical behavior of its generating functions.

1. Introduction

Let (X,d) be a compact metric space and let $\mathbb{F} = \{f_n : n \in \mathbb{N}\}$ be a family of continuous self-maps on X. Any such family $\mathbb{F}$ generates a non-autonomous dynamical system via the relation $x_n = f_n(x_{n-1})$; such a dynamical system will be denoted by $(X,\mathbb{F})$. For any $x \in X$, $\{f_n \circ f_{n-1} \circ \ldots \circ f_1(x) : n \in \mathbb{N}\}$ defines the orbit of x. The objective of study of a non-autonomous dynamical system is to investigate the orbit of an arbitrary point x in X. For notational convenience, let $\omega_n(x) = f_n \circ f_{n-1} \circ \ldots \circ f_1(x)$ be the state of the system after n iterations. If $y = \omega_n(x) = f_n \circ f_{n-1} \circ \ldots \circ f_1(x)$, then $x \in f_1^{-1} \circ f_2^{-1} \circ \ldots \circ f_n^{-1}(y) = \omega_n^{-1}(y)$ and, hence, ω_n^{-1} traces the point n units back in time.

A point x is called *periodic* for $\mathbb{F}$ if there exists $n \in \mathbb{N}$ such that $\omega_{nk}(x) = x$ for all $k \in \mathbb{N}$. The least such n is known as the period

2010 *Mathematics Subject Classification.* 37B20, 37B55, 54H20.
Key words and phrases. Li-Yorke chaos, non-autonomous dynamical systems, topological entropy, topological mixing, transitivity, weakly mixing.