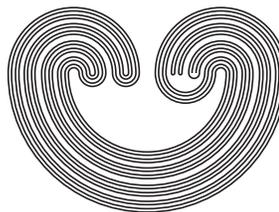


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## A MONOTONICALLY RETRACTABLE REALCOMPACT SPACE WHICH IS NOT LINDELÖF

by

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## A MONOTONICALLY RETRACTABLE REALCOMPACT SPACE WHICH IS NOT LINDELÖF

MASAMI SAKAI

**ABSTRACT.** We construct a monotonically retractable realcompact space which is not Lindelöf. This answers a question posed by R. Rojas-Hernández in *Function spaces and D-property* [Topology Proc. **43** (2014)].

### 1. INTRODUCTION

Throughout this paper, all spaces are assumed to be Tychonoff. For a set  $S$ ,  $[S]^{\leq \omega}$  stands for the set of countable subsets in  $S$ . A space having a countable network is said to be *cosmic*, where a family  $\mathcal{N}$  of subsets of a space  $X$  is said to be a *network* for  $X$  if for any  $x \in X$  and any neighborhood  $U$  of  $x$ , there exists some  $N \in \mathcal{N}$  such that  $x \in N \subset U$ .

For a space  $X$ , let  $C_p(X)$  be the space of all real-valued continuous functions of  $X$  with the topology of pointwise convergence. For each  $n \in \mathbb{N}$ , let  $C_{p,n}(X)$  be the  $n$ -times iterated function space of  $X$ . G. A. Sokolov ([9], [10]) proved that  $C_{p,n}(K)$  of a Corson compact space  $K$  is Lindelöf for each  $n \in \mathbb{N}$ . Motivated by Sokolov's result, Vladimir V. Tkachuk introduced the following.

**Definition 1.1** ([11]). A space  $X$  is *Sokolov* if for any sequence  $\{F_n : n \in \mathbb{N}\}$  with  $F_n$  closed in  $X^n$ , there exists a continuous map  $f : X \rightarrow X$  such that  $f(X)$  is cosmic and  $f^n(F_n) \subset F_n$  for each  $n \in \mathbb{N}$ .

A Corson compact space is Sokolov, and all the spaces  $C_{p,n}(X)$  are Lindelöf for a Sokolov space  $X$  with an additional condition [11, Theorem 2.1]. A Sokolov space is collectionwise normal,  $\omega$ -monolithic (i.e., the

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