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A MONOTONICALLY RETRACTABLE REALCOMPACT SPACE WHICH IS NOT LINDELÖF

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Abstract. We construct a monotonically retractable realcompact space which is not Lindelöf. This answers a question posed by R. Rojas-Hernández in *Function spaces and D-property* [Topology Proc. **43** (2014)].

1. Introduction

Throughout this paper, all spaces are assumed to be Tychonoff. For a set S, $[S]^{\leq\omega}$ stands for the set of countable subsets in S. A space having a countable network is said to be *cosmic*, where a family $\mathcal{N}$ of subsets of a space X is said to be a *network* for X if for any $x \in X$ and any neighborhood U of x, there exists some $N \in \mathcal{N}$ such that $x \in N \subset U$.

For a space X, let $C_p(X)$ be the space of all real-valued continuous functions of X with the topology of pointwise convergence. For each $n \in \mathbb{N}$, let $C_{p,n}(X)$ be the n-times iterated function space of X. G. A. Sokolov ([9], [10]) proved that $C_{p,n}(K)$ of a Corson compact space K is Lindelöf for each $n \in \mathbb{N}$. Motivated by Sokolov's result, Vladimir V. Tkachuk introduced the following.

Definition 1.1 ([11])**.** A space X is *Sokolov* if for any sequence $\{F_n : n \in \mathbb{N}\}$ with F_n closed in X^n, there exists a continuous map $f : X \to X$ such that $f(X)$ is cosmic and $f^n(F_n) \subset F_n$ for each $n \in \mathbb{N}$.

A Corson compact space is Sokolov, and all the spaces $C_{p,n}(X)$ are Lindelöf for a Sokolov space X with an additional condition [11, Theorem 2.1]. A Sokolov space is collectionwise normal, ω-monolithic (i.e., the

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