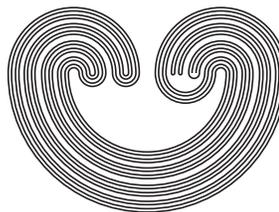


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## PATCHWORK AND WEB SPACES

by

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## PATCHWORK AND WEB SPACES

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**ABSTRACT.** Patch topologies are obtained by joining a given topology with a second topology having the dual specialization order. They provide a convenient passage from topological spaces to semi-qospaces (i.e., quasi-ordered sets equipped with a topology, making principal ideals and filters closed) that have better separation properties than the original spaces. Web spaces, originally defined by the existence of neighborhood bases consisting of webs (where a web around  $x$  contains with any  $y$  a lower bound of  $x$  and  $y$ ), may be characterized by the condition that the interior operator preserves finite unions of saturated sets. More important in the patch game is that the web spaces are precisely those for which any patch space determines the original open sets as the upper sets generated by the patch open sets. Via suitable patch functors, the category of web spaces is concretely isomorphic to various categories of strongly convex web semi-qospaces. We apply the patch construction to semitopological semilattices (as specific web ordered spaces) and show that the  $T_0$  web spaces are exactly the so-called  $\downarrow$ -consistent subspaces of semitopological semilattices with a compatible topology; similar representations are established for web ordered spaces. A look at regularity axioms for patch spaces concludes the study.

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