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by

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UNIVERSAL BOUND ON THE MEASURE OF PERIODIC WINDOWS OF PARAMETERIZED CIRCLE MAPS

SUDDHASATTWA DAS

Abstract. One-parameter families f_t of circle diffeomorphisms are a common occurrence in dynamical systems. One subject of investigation is the variation of the rotation number with the parameter and how the parameter-range splits into periodic windows and a Cantor set of irrational rotation numbers. One of the earliest topics of investigation is how the measure of this Cantor set depends on the family, starting with the work of V. I. Arnol'd, Michael-Robert Herman, etc. Studies of various parameterized circle maps seem to indicate that this measure approaches 1 when f_t is a small perturbation of the identity, and sometimes approaches 0 when f_t is close to a critical map. This paper describes a universal function η which gives an upper bound on the Lebesgue measure of the periodic windows based on the C^4 distance of f_t from the identity map. This confirms several observations made in the mathematical literature in the past.

1. Introduction and Main Results

In this paper, the unit circle S^1 will be identified as $\mathbb{R}/\mathbb{N}$, and $proj : \mathbb{R} \to \mathbb{N}$ is the associated quotient map. A homeomorphism of the circle $f : S^1 \to S^1$ can be lifted to a map $\bar{f} : \mathbb{R} \to \mathbb{R}$ under the covering map $proj$. It is well known (see, for example, [13]) that the following limit exists and is a constant independent of z.

$$\rho(f) := \lim_{n\to\infty} \frac{\bar{f}(z) - z}{n}$$

This limit is called the *rotation number* of f. The rotation number is of fundamental importance in inferring the properties of the map and its

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