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by

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# ON ORDERED TOPOLOGICAL VECTOR SPACES WITH POSITIVE INTERIOR POINTS

KAORI YAMAZAKI

Abstract. Answering a question indicated by Er-Guang Yang in (*Partial answers to some questions on maps to ordered topological vector spaces*, Topology Proc. **50** (2017), 311–317), we show that, for an ordered topological vector space $Y$ with positive interior points, if each non-zero positive element is an order unit, then $Y$ is isomorphic to the real line. We also provide a technique which reduces some vector-valued results to the original real-valued ones by using some Minkowski functionals.

## 1. Introduction

Throughout this paper, let $\mathbb{R}$ be the set of all real numbers, and $\mathbb{N}$ the set of all natural numbers.

Let us recall some terminology from [1] and [5]. A partially ordered real vector space $(Y, \leq)$ is said to be an *ordered vector space* if the following conditions are satisfied:

(i) $x \leq y$ implies $x + z \leq y + z$ for all $x, y, z \in Y$,
(ii) $x \leq y$ implies $rx \leq ry$ for all $x, y \in Y$ and all $r \in \mathbb{R}$ with $r \geq 0$.

Let $(Y, \leq)$ be an ordered vector space. Then, $y \in Y$ is *positive* if $\mathbf{0} \leq y$, and the set $\{y \in Y : \mathbf{0} \leq y\}$, called the *positive cone* of $Y$, is denoted by $Y^+$. For $y_1, y_2 \in Y$ with $y_1 \leq y_2$, the subspace $(y_1 + Y^+) \cap (y_2 - Y^+)$ of $Y$, called an *order interval*, is denoted by $[y_1, y_2]$. A topological vector space $Y$ is called an *ordered topological vector space* (*o.t.v.s.*) if $Y$ is an ordered vector space such that the positive cone $Y^+$ is closed in $Y$. It is

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