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CLASSIFICATION OF SIMPLY-CONNECTED TRIVALENT 2-DIMENSIONAL STRATIFOLDS

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Abstract. Trivalent 2-stratifolds are a generalization of 2-manifolds in that there are disjoint simple closed curves where locally three sheets meet. We obtain a classification of trivalent 1-connected 2-stratifolds in terms of their associated labeled graphs.

1. Introduction

A closed 2-*stratifold* is a 2-dimensional cell complex X that contains a collection of finitely many simple closed curves, the components of the 1-skeleton $X^{(1)}$ of X, such that $X - X^{(1)}$ is a 2-manifold and a neighborhood of each point in a component C of $X^{(1)}$ consists of $n \geq 3$ sheets. 2-stratifolds X are a more restricted class than multibranched surfaces, studied by Shosaka Matsuzaki and Makoto Ozawa in [9], and trivalent 2-stratifolds (defined in §2) are a more restrictive class than foams, which have been studied by J. Scott Carter [2] and Mikhail Khovanov [8]. Foams include special spines S that occur as spines of (closed) 3-manifolds M (see, for example, [10] and [12]). Thus, $\pi_1(M) \cong \pi_1(S)$ for some special spine S. But there are significant differences between the fundamental groups of 2-stratifolds and 3-manifolds: 3-manifold groups are residually finite, but every Baumslag–Solitar group (some Hopfian, others non-Hopfian) can be realized as the fundamental group of a 2-stratifold. Also, with the exception of lens spaces and connected sums, closed 3-manifolds are determined by their fundamental groups (see, for example, [1]), but there are infinitely many non-homeomorphic 2-stratifolds with the same fundamental group. However, it can be shown that fundamental groups

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