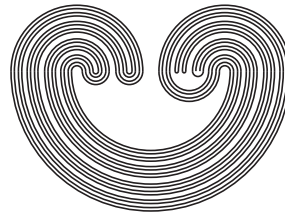


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ON A CONSTRUCTION OF MALYKHIN

by

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ON A CONSTRUCTION OF MALYKHIN

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ABSTRACT. We construct, using \diamond , a nondiscrete Hausdorff extremally disconnected topological group of size ω_1 where every countable subset is closed and discrete.

1. INTRODUCTION

A topological space is called *extremally disconnected* [8], if the closure of any open set in this space is open (or, equivalently, the closures of any two disjoint open sets are disjoint). In 1967, Arhangel'skii posed the problem of the existence in ZFC of a nondiscrete Hausdorff extremally disconnected topological group [1]. Recently Reznichenko and Sipacheva in [6] have announced a proof that the existence of a countable nondiscrete Hausdorff extremally disconnected group implies the existence of a rapid ultrafilter; hence, such a group cannot be constructed in ZFC because the nonexistence of rapid ultrafilters is consistent with ZFC (see [5]). The general case is still open. In fact, the uncountable version of Arhangel'skii's problem remains largely unexplored. Among the few results that exist, we can find a forcing construction of Malykhin of a nondiscrete Hausdorff extremally disconnected group in which all countable subsets are closed and discrete [4]. More explicitly, he introduced a σ -close forcing notion

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