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INVERSE LIMITS WITH SET-VALUED FUNCTIONS HAVING GRAPHS THAT ARE SINUSOIDS

W. T. INGRAM

Abstract. Sinusoids are subcontinua of the unit square similar to the standard $\sin(1/x)$-curve. We show that inverse limits with a sequence of set-valued functions having graphs that are sinusoids are chainable continua. In the process we prove that if M and N are continua and $f : M \twoheadrightarrow N$ is a monotone mapping such that point inverses under f are C-sets with property P and the continuum N has property P then M has property P where P is any one of the properties: (1) atriodic, (2) hereditarily decomposable, (3) hereditarily unicoherent, and (4) hereditarily decomposable chainable.

1. Introduction

In 1955 A. D. Wallace introduced the notion of a C-set and explored C-sets in the context of topological semigroups. In 1982 the author [4] investigated C-sets and their role in continuum theory. In the present article, we make use of C-sets to obtain results about inverse limits with upper semi-continuous bonding functions. Specifically, we show in Theorem 4.3 that inverse limits on $[0, 1]$ with upper semi-continuous set-valued bonding functions having graphs that are sinusoids are chainable continua. In an earlier paper, [6, Example 5.4], we showed that the inverse limit on $[0, 1]$ with an upper semi-continuous bonding function whose graph is a piecewise linear version of a "standard" $\sin(1/x)$-curve is a chainable continuum. Soon thereafter James P. Kelly, [11], extended that result to inverse limits on $[0, 1]$ with a single upper semi-continuous function having a graph that is what he calls an irreducible function.

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