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by

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ON H-CLOSED AND MINIMAL HAUSDORFF SPACES AND THE BOOLEAN PRIME IDEAL THEOREM

ELEFTHERIOS TACHTSIS

Abstract. In ZF (i.e., Zermelo–Fraenkel set theory without the Axiom of Choice (AC)), we establish that the *Boolean Prime Ideal Theorem* (BPI) is equivalent to each one of the following statements:

(1) A Hausdorff space is H-closed if and only if every open ultrafilter on the space converges;

(2) Products of H-closed Hausdorff spaces are H-closed;

(3) Products of minimal Hausdorff spaces are minimal;

(4) For every Hausdorff space X, the Katětov space κX is an H-closed extension of X;

(5) Every Hausdorff space has a (unique up to homeomorphism) projectively largest Katětov H-closed extension.

We also establish the following implications: BPI $\Rightarrow$ "products of non-empty H-closed Hausdorff spaces are non-empty" $\Rightarrow$ "products of non-empty minimal Hausdorff spaces are non-empty" $\Rightarrow$ $\mathsf{AC}_{\mathsf{fin}}$ (i.e., "every family of non-empty finite sets has a choice function").

1. Introduction

An *extension* of a topological space X is a space which contains X as a dense subspace. The construction of extensions such as compactifications, realcompactifications and H-closed extensions has been an area of intense research in general topology for a long time. For a systematic and deep study of extensions (and absolutes) of Hausdorff spaces the reader is referred to the book of Porter and Woods [20].

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