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Distance Domains: Completeness

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DISTANCE DOMAINS: COMPLETENESS

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Abstract. We explore extensions of domain theoretic concepts, replacing transitive relations with general non-symmetric distances. These lead to a generalization of Smyth-completeness which we characterize in various ways analogous to our previous Yoneda-completeness characterizations.

Motivation

A number of works have extended domain theory from posets to more metric-like structures (see [8]). However, both the classical theory and these generalizations tend to focus on just one aspect of the dual nature of domains. Our primary goal is explore the other aspect.

More precisely, the standard approach to domain theory is to start with a partial order $\leq$ and then define its way-below relation $\ll$, a transitive but generally non-reflexive relation. An alternative approach is to start with a transitive relation $\ll$ and then define its lower order $\leq$. Using maxima rather than suprema, one also obtains dual notions of completeness and continuity for $\ll$. This is the approach we generalize, working with a general non-symmetric distance $\mathbf{d}$ and its lower hemimetric $\underline{\mathbf{d}}$.

Also, previous works have developed quantitative domain theory in a highly category or fuzzy theoretic way (see, e.g., [10] and [15]). Another goal of our paper is to provide a more classic approach through topology, metric and order theory, building on [9]. This leads to certain natural generalizations and should also be more accessible to analysts.

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