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ON PERIODIC DATA OF DIFFEOMORPHISMS WITH ONE SADDLE ORBIT

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Abstract. In this paper we find all possible periodic data for orientation preserving gradient-like diffeomorphisms of orientable surfaces with one saddle orbit. We also construct a system of this class for every admissible collection of periodic data.

1. Introduction

In the study of discrete dynamical systems, i.e., the study of orbits of self-maps f defined on a given compact manifold, the periodic behavior plays an important role. In the last forty years there was a growing number of results showing that certain simple hypotheses force qualitative and quantitative properties (like the set of periods) of a system. One of the best-known results is the title of the paper "Period three implies chaos for the interval continuous self-maps" [11]. The effect described there was discovered by O. M. Šarkovs'kiĭ in [14]. The most useful tools for proving the existence of fixed points or, more generally, of periodic points for a continuous self-map f of a compact manifold is the Lefschetz fixed point theorem and its improvements (see, for instance [3] and [4]). The Lefschetz zeta-function simplifies the study of the periodic points of f. This is a generating function for all the Lefschetz numbers of all iterates of f.

The periodic data of diffeomorphisms with regular dynamics on surfaces were studied by means zeta-function in a series of already classical works by such authors as Paul R. Blanchard, John M. Franks, Rufus Bowen, Steve Batterson, John Smillie, William H. Jaco, Peter B. Shalen, Carolyn C. Narasimhan, and others. A description of periodic data of gradient-like diffeomorphisms of surfaces was given in [1] by means of classification of periodic surface transformations obtained by Jakob Nielsen [12].

In [8], the authors show that the study of periodic data of arbitrary Morse–Smale diffeomorphisms on surfaces is reduced by filtration to the problem of computing

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