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Abstract. In this paper we make some observations concerning $M-$metric spaces and point out some discrepancies in some proofs found in the literature. To remedy this, we propose a new topological construction and prove that it is, in fact, a generalization of a partial metric space. Then, using this construction, we present our main theorem, having as its corollaries the fixed point theorems found in previous publications.

1. Introduction

In 2014, Mehdi Asadi, Erdal Karapinar, and Peyman Salimi [1] proposed the $M-$metric, an intended generalization of a partial metric. In their paper, the proof of Lemma 2.5 does not hold, as we demonstrate in Example 2.4. Although it is a small lemma, its assertion was crucial to the proof of their main theorems: Theorem 3.1 and Theorem 3.2. Our main concern in their approach lies in the open balls they proposed. We go more in depth on the subject in §4.

In §2, we introduce the $M-$metric presented in [1] and generalize it to allow negative values. We also present examples that show why some assumptions proposed in [1], including Lemma 2.5, are not accurate.

In §3, we present the partial metric found in [2], [6], and [7]. We also show how to induce a partial metric from an $M-$metric. The purpose of this section is to put in perspective the generalization from a partial metric to an $M-$metric.

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Key words and phrases. fixed point theorem, generalized metric, $M-$metric, partial metric.