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by

Mathieu Baillif

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings
	Department of Mathematics & Statistics
	Auburn University, Alabama 36849, USA
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NOTES ON LINEARLY H-CLOSED SPACES AND OD-SELECTION PRINCIPLES

MATHIEU BAILLIF

ABSTRACT. A space is called *linearly H-closed* if and only if any chain cover possesses a dense member. This property lies strictly between feeble compactness and H-closedness. While regular H-closed spaces are compact, there are non-compact linearly H-closed spaces which are even collectionwise normal and Fréchet–Urysohn. We give examples in other classes and ask whether there is a first countable normal linearly H-closed non-compact space in ZFC. We show that PFA implies a negative answer if the space is moreover either locally separable or both locally compact and locally ccc. An Ostaszewski space (built with $\diamondsuit$) is an example which is even perfectly normal. We also investigate Menger-like properties for the class of od-covers, that is, covers whose members are open and dense.

1. INTRODUCTION

This note is mainly about a property (to our knowledge not investigated before) we decided to call linear H-closedness, which lies strictly between H-closedness and feeble compactness. Since it came up while investigating simple instances of od-selection properties (see below), and all have a common "density of open sets" flavor, we include a section about this latter topic although they are not related more than on a superficial level.

By "space" we mean "topological space." We take the convention that "regular" and "normal" imply "Hausdorff." A *cover* of a space always means a cover by open sets, and a cover is a *chain cover* if it is linearly ordered by the inclusion relation. In any Hausdorff space (of cardinality

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