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RIGHT-ANGLED MOCK REFLECTION SURFACES

RICHARD SCOTT AND TIMOTHY SCHROEDER

ABSTRACT. In articles by M. Davis, T. Januskiewicz, and R. Scott (see *Nonpositive curvature of blow-ups*, Selecta Math. (N.S.) **4** (1998), no. 4, 491–547 and *Fundamental groups of blow-ups*, Adv. Math. **177** (2003), no. 1, 115–179) and another article by Richard Scott (see *Right-angled mock reflection and mock Artin groups*, Trans. Amer. Math. Soc. **360** (2008), no. 8, 4189–4210), a generalization of right-angled Coxeter groups, so-called right-angled mock reflection groups (RAMRGs), are introduced and explored. As is the case with Coxeter groups, these groups can be defined by a finite simple graph (one that is now "decorated" in the language of Colin Hagemeyer and Richard Scott in *On groups with Cayley graph isomorphic to a cube* [Comm. Algebra **42** (2014), no. 4, 1484–1495]), and they act on CAT(0) cubical complexes such that the stabilizer of every edge is $\mathbb{Z}_2$ and the 1-skeleton of the link of every vertex is isomorphic to the defining graph. This note examines the case where the defining graph Γ is homeomorphic to $\mathbb{S}^1$, and thus the corresponding cubical complex Σ is a 2-manifold. We show, explicitly, that these RAMRGs are virtually torsion-free and we describe the resulting quotient surfaces.

1. Introduction

Let Γ be a finite, simple graph with vertex set S. Γ encodes the data for a presentation of a *right-angled Coxeter group* (*RACG*) W_Γ:

$$W_\Gamma = \left\langle S \mid s^2 = 1 \text{ for each } s \in S \text{ and } (st)^2 = 1 \text{ for each edge } \{s,t\} \text{ of } \Gamma \right\rangle.$$

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