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TOPOLOGY PROCEEDINGS

Volume 54, 2019

Pages 205–231

http://topology.nipissingu.ca/tp/

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Electronically published on March 26, 2019

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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Volume 54 (2019)
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E-Published on March 26, 2019

EDGE PRESERVING MAPS OF THE CURVE GRAPHS IN LOW GENUS

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Abstract. Let R be a compact, connected, orientable surface of genus g with n boundary components. Let $\mathcal{C}(R)$ be the curve graph of R. We prove that if $g = 0$, $n \geq 5$ or $g = 1$, $n \geq 3$, and $\lambda : \mathcal{C}(R) \to \mathcal{C}(R)$ is an edge preserving map, then λ is induced by a homeomorphism of R, and this homeomorphism is unique up to isotopy.

1. Introduction

Let R be a compact, connected, orientable surface of genus g with n boundary components. The *mapping class group*, Mod_R, of R is defined to be the group of isotopy classes of orientation preserving self-homeomorphisms of R. The *extended mapping class group*, Mod_R^*, of R is defined to be the group of isotopy classes of all self-homeomorphisms of R. Abstract simplicial complexes on surfaces have been studied to get information about the algebraic structure of the extended mapping class groups of the surfaces. One of these complexes is the complex of curves. The vertex set of the complex of curves is the set of isotopy classes of nontrivial simple closed curves on R, where nontrivial means the curve does not bound a disk and it is not isotopic to a boundary component of R. A set of vertices forms a simplex in the complex of curves if its elements can be represented by pairwise disjoint simple closed curves on the surface. Let $\mathcal{C}(R)$ be the curve graph, the first skeleton of the complex of curves on R. A map on $\mathcal{C}(R)$ is edge preserving if it sends two vertices

2010 *Mathematics Subject Classification.* 20F65, 57M07.

Key words and phrases. edge preserving maps, mapping class groups, orientable surfaces.