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HOMOLOGICALLY EQUIVALENT DISCRETE MORSE FUNCTIONS

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Abstract. A theory of homological equivalence of discrete Morse functions is developed in this paper, extending the work of R. Ayala, L. M. Fernández, D. Fernández-Ternero, and J. A. Vilches [*Discrete Morse theory on graphs*, Topology Appl. **156** (2009), no. 18, 3091–3100] and Ayala, Fernández, and J. A. Vilches [*Characterizing equivalent discrete Morse functions*, Bull. Braz. Math. Soc. (N.S.) **40** (2009), no. 2, 225–235]. This is accomplished by defining the homological sequence associated with a discrete Morse function on any finite simplicial complex and developing its basic properties. These properties allow us to show that certain homological sequences may be viewed as lattice walks satisfying parameters. We count the number of discrete Morse functions up to homological equivalence on all collapsible 2-dimensional complexes by constructing discrete Morse functions inducing the desired sequence. The paper concludes with an example to illustrate our construction.

1. Introduction

Discrete Morse theory was invented by Robin Forman [6] as an analogue of "smooth" Morse theory popularized by J. Milnor [10]. Many classical results in Morse theory, such as the Morse inequalities, carry over into the discrete setting [8]. Applications of discrete Morse theory are vast,

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