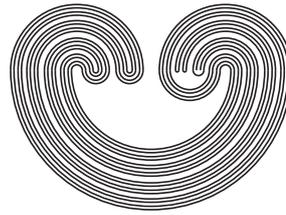


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TOPOLOGY PROCEEDINGS



Volume 55, 2020

Pages 35–38

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by

HELGE GLÖCKNER AND NIKU MASBOUGH

Electronically published on February 14, 2019

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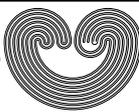
Web: <http://topology.auburn.edu/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

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PRODUCTS OF REGULAR LOCALLY COMPACT SPACES ARE $K_{\mathbb{R}}$ -SPACES

HELGE GLÖCKNER AND NIKU MASBOUGH

ABSTRACT. A theorem by N. Noble from 1970 asserts that every cartesian product of completely regular, locally pseudo-compact $k_{\mathbb{R}}$ -spaces is a $k_{\mathbb{R}}$ -space. As a consequence, all products of locally compact Hausdorff spaces are $k_{\mathbb{R}}$ -spaces. We provide an accessible proof for this fact. More generally, we show that all products of not necessarily Hausdorff, regular locally compact spaces are $k_{\mathbb{R}}$ -spaces.

1. INTRODUCTION

Recall that a (not necessarily Hausdorff) topological space X is called *compact* if every open cover of X has a finite subcover, and *locally compact* if every neighborhood of a point in X contains a compact neighborhood. If every neighborhood contains a closed neighborhood, then X is called *regular*; it is *completely regular* if the topology on X is initial with respect to the set $C(X, \mathbb{R})$ of all continuous real-valued functions on X . A function $f: X \rightarrow Y$ to a topological space Y is called *k -continuous* if its restriction $f|_K: K \rightarrow Y$ is continuous for each compact subset $K \subseteq X$. If every k -continuous real-valued function on X is continuous, then X is called a *$k_{\mathbb{R}}$ -space*. Every k -continuous function from a $k_{\mathbb{R}}$ -space to a completely regular topological space is continuous. We shall always endow cartesian products of topological spaces with the product topology. Stimulated by [4], we prove the following theorem:

2010 *Mathematics Subject Classification.* Primary 54B10; Secondary 54D45, 54D50.

Key words and phrases. Compactly generated space, cartesian product, product topology, locally compact space, regular space, Hausdorff property.

The first author was supported by DFG grant GL 357/9-1.

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