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by

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SPACES WITH NO S OR L SUBSPACES

JOAN E. HART AND KENNETH KUNEN

Abstract. We consider spaces that contain neither an S-space nor an L-space. We call such a space ESLC, and show that it is consistent with ZFC for a product of two ESLC spaces to contain both an S-space and an L-space.

1. Introduction

All topological spaces considered in this paper are T_3 (Hausdorff and regular).

The following terminology is standard: A space X is *hereditarily separable* (*HS*) iff all subspaces of X are separable, and *hereditarily Lindelöf* (*HL*) iff all subspaces of X are Lindelöf. Then, X is an *S-space* iff X is HS but not HL, and X is an *L-space* iff X is HL but not HS.

S-spaces are consistent with $\mathsf{MA}(\aleph_1)$ [14], but are refuted by PFA [16], while L-spaces exist in ZFC [10]. Under CH, a large variety of S-spaces and L-spaces have been constructed, and still more have been built under $\diamondsuit$.

In this paper, we shall study the following notion:

Definition 1.1. The space X is *ESLC* iff every subspace of X is either both HS and HL or neither HS nor HL.

This is the same as saying that no subspace of X is either an S-space or an L-space. The reason for the "C" in "ESLC" is the following slightly non-standard terminology for a standard concept:

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