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by

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COMPLETE REGULARITY *À LA QUANTALE*

JORGE BRUNO

Abstract. Nearly three decades from his celebrated result, we study a modern refinement and strengthening of Kopperman's full metrisabilty of all topological spaces. Within this new theory of V-*spaces*, developed by Flagg and Weiss, we investigate several topological notions and their metric counterparts. Among our main results is the reconstruction, in terms of V-spaces, of Kopperman's equivalence between symmetric continuity spaces valued in a value semigroup and completely regular topologies. We conclude our work by revisiting some classical topological results and their almost evident validity through this metric lens.

1. Introduction

Let **Met** denote the of category metric spaces with ϵ-δ continuous functions. The coincidence of both types of morphisms implies that the obvious functor $\mathbf{Met} \to \mathbf{Top}$, that sends any metric space to the topological space it generates, is fully faithful on $\mathbf{Top_M}$ - the category of metrisable topological spaces. The fullness of this functor is largely due to the triangle inequality imposed on any metric space. Were it not the case, it would only be true that ϵ-δ continuity implies topological continuity. Indeed, take $X = \{a, b, c, e\}$ with $d : X^2 \to \mathbb{R}$ as

$$d(x,y) = \begin{cases} 0 & \text{if } \{x,y\} = \{a,b\} \text{ or } \{x,y\} = \{b,c\}, \\ 2 & \text{if } \{x,y\} = \{a,c\}, \text{ and} \\ 1 & \text{otherwise.} \end{cases}$$

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